3. QUADRATIC FUNCTIONS

IMPORTANT NOTES:

(i) The general form of a quadratic function is $f(x) = ax^2 + bx + c$; a, b, c are constants and $a \neq 0$.
(ii) Characteristics of a quadratic function:
  (a) Involves one variable only,
  (b) The highest power of the variable is 2.

3.1.1 Recognising a quadratic function

EXAMPLE

No.	Quadratic Functions	Non-Quadratic Function	Reason
1.	$f(x) = x^2 + 2x - 3$	$f_2(x) = 2x - 3$	No terms in x^2 (a = 0)
2.	$g(x) = x^2 - ½$	$g(x) = x^2 + \frac{2}{x}$	The term $\frac{2}{x}$
3.	$h(x) = 4 - 3x^2$	$h(x) = x^3 - 2x^2$	The term x^3
4.	$y = 3x^2$	$y = 3x^{-2}$	The term x^{-2}
5.	$p(x) = 3 - 4x + 5x^2$	$x^2 - 2xy + y^2$	Two variables

Exercise : State whether the following are quadratic functions. Give your reason for Non Q.Functions.

No.	Functions	Q.F.	Non-Q.F.	REASON
0.	$f(x) = 10$		√	No terms in x^2 (second degree)
1.	$f(x) = 10^2$			
2.	$g(x) = 10 - x^2$			
3.	$p(x) = x^2 + x$			
4.	$y = 2x^2 + ½ x - 3$			
5.	$y = -\frac{6}{x}$			
6.	$f(x) = x(x - 2)$			
7.	$g(x) = 2x^2 + kx - 3$, k a constant			
8.	$h(x) = (m-1)x^2 + 5x + 2m$, m constant			
9.	$y = 3 - (p+1)x^2$, p constant			
10.	$p(x) = x^2 + 2hx + k + 1$, h, k constants			

NOTE **: The proper way to denote a quadratic function is** $f : x \to ax^2 + bx + c$**.**
$f(x) = ax^2 + bx + c$ is actually the value (or image) f for a given value of x.

3.2 Minimum Value and Maximum value of a Quadratic Function

Do you know that ....

A non-zero number when squared will always be positive ?

$3^2 = 9$ $\quad (-5)^2 = 25$ $\quad (\frac{1}{2})^2 = \frac{1}{4}$ $\quad (-1)^2 = 1$

So, what is the minimum value when we find the square of a number ?

Minimum value of x^2 is ..... 0 !
This is obtained when $x = 0$.

$(\quad)^2 = 0$

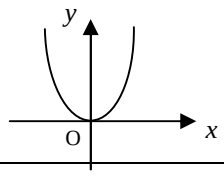
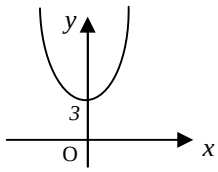
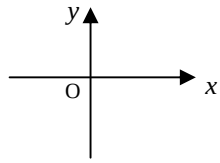
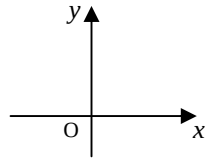
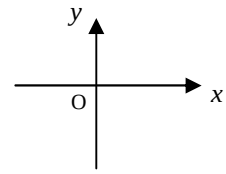
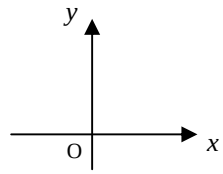
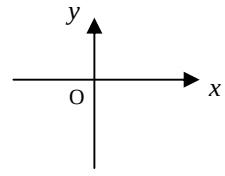
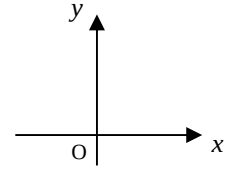
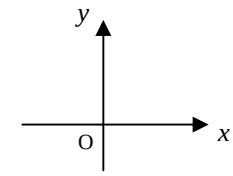
The value inside the brackets must be 0 !

So, the minimum value of x^2 is 0;
the minimum value of $x^2 + 3$ is $0 + 3 = 3$
the minimum value of $x^2 - 8$ is $0 + (-8) = -8$
the minimum value of $x^2 + 100$ is $0 + 100 = 100$

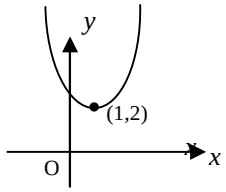
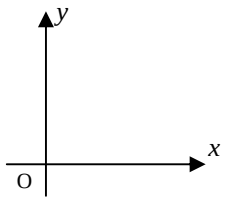
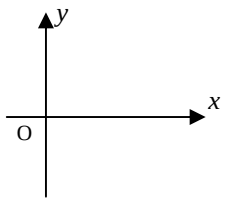
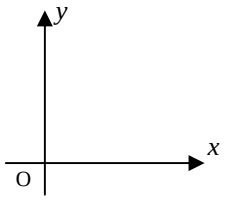
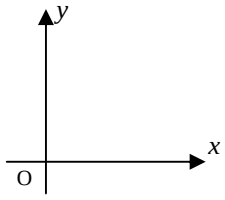
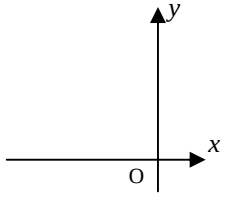
The minimum value of x^2 is 0 ,
It means $x^2 \geq 0$,
So, $-x^2 \leq 0$

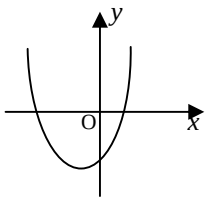
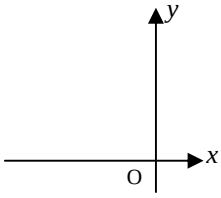
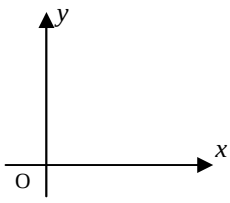
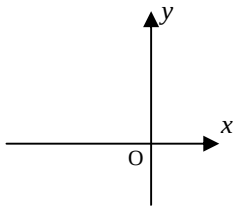
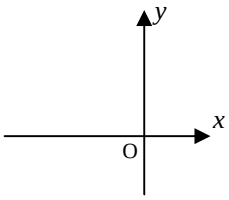
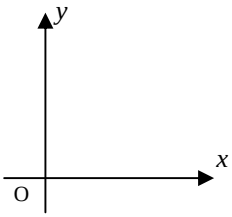
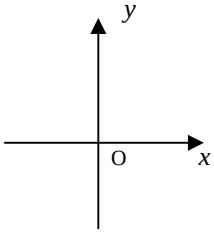
Hence the maximum value of $-x^2$ is 0
the maximum value of $-x^2 + 5$ is 5
the maximum value of $-x^2 - 3$ is -3

3.2.1 To state the **Minimum value of a Quadratic Function $f(x) = ax^2 + c$, $a > 0$**

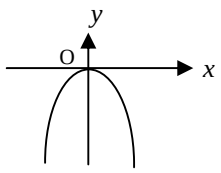
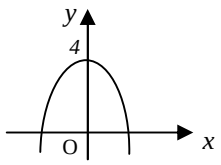
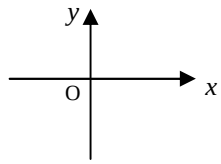
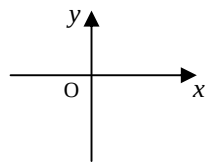
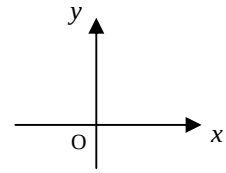
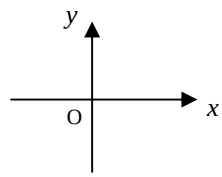
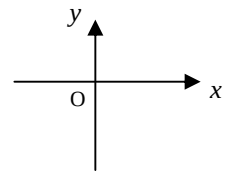
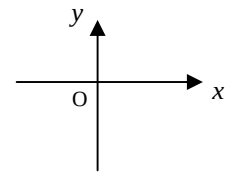
No.	Function	Minimum value of y	Corresponding value of x	Minimum Point	(Sketched) Graph
1.	$f(x) = x^2$	0	$x = 0$	(0, 0)	
2.	$g(x) = x^2 + 3$	3			
3.	$h(x) = \boldsymbol{x^2 - 4}$		**0**	**(0, -4)**	
4.	$y = x^2 + ½$				
5.	$p(x) = x^2 - 10$				
6.	$f(x) = 2x^2 + 3$				
7	$g(x) = ½ x^2 - 5$				
8.	$h(x) = 10x^2 + 1$				
9.	$y = 4 + 2x^2$				

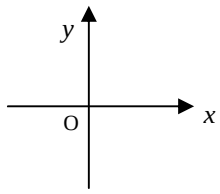
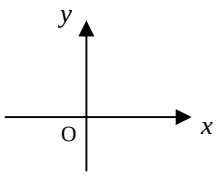
3.2.2 To state the Minimum Value of a Quadratic Function in the form $f(x) = a(x + p)^2 + q$, $a > 0$

No.	Function	Minimum value of y	Corresponding value of x	Minimum Point	(Sketched) Graph
1.	$f(x) = (x - 1)^2 + 2$	2	$(x - 1)^2 = 0$ $x = 1$	(1, 2)	
2.	$g(x) = (x - 2)^2 + 4$	4	$(x - 2)^2 = 0$ $x =$	(,)	
3.	$h(x) = (x - 1)^2 - 3$				
4.	$y = (x - 2)^2$				
5.	$f(x) = (x - 3)^2 + 2$				
6.	$f(x) = (x + 2)^2 + 3$				

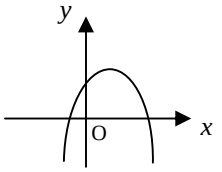
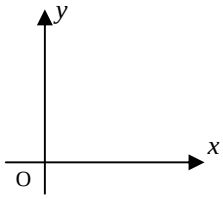
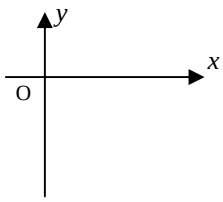
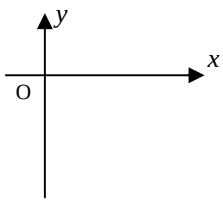
No.	Function	Minimum value of y	Corresponding value of x	Minimum Point	(Sketched) Graph
7.	$f(x) = (x + 1)^2 - 4$				
8.	$f(x) = 2(x + 3)^2$				
9.	$f(x) = 2(x - 1)^2 + 3$				
10.	$f(x) = 3(x + 2)^2 - 1$				
11.	$f(x) = 2 + (x + 1)^2$				
12.	$f(x) = 1 + 2(x - 3)^2$				
13.	$f(x) = 3x^2 - 2$				

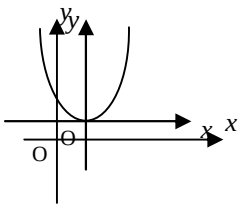
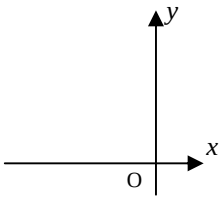
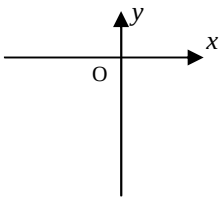
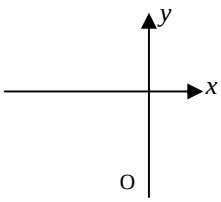
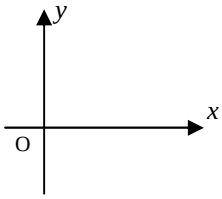
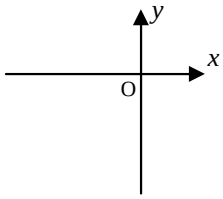
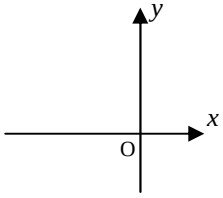
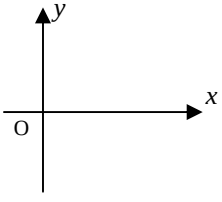
3.2.3 To state the Maximum Value of a Quadratic Function in the form $f(x) = ax^2 + c$, $a < 0$

No.	Function	Maximum value of y	Corresponding value of x	Maximum Point	(Sketched) Graph
1.	$f(x) = -x^2$	0	$x = 0$	(0, 0)	
2.	$g(x) = -x^2 + 4$	4			
3.	$h(x) = -\mathbf{x^2 + 2}$		**0**	**(0, 2)**	
4.	$y = -x^2 + ½$				
5.	$p(x) = 9 - x^2$				
6.	$f(x) = -2x^2 + 3$				
7	$g(x) = -½ x^2 - 1$				
8.	$h(x) = 2 - 10x^2$				

No.	Function	Minimum value of y	Corresponding value of x	Minimum Point	(Sketched) Graph
9.	$y = 4 - 2x^2$				
10.	$p(x) = 5 - 3x^2$				

3.2.5 To state the Maximum Value of a Quadratic Function in the form $f(x) = a(x + p)^2 + q$, $a < 0$

No.	Function	Maximum value of y	Corresponding value of x	Maximum Point	(Sketched) Graph
1.	$f(x) = -(x - 1)^2 + 2$	2	$(x - 1)^2 = 0$ $x = 1$	(1, 2)	
2.	$g(x) = -(x - 2)^2 + 4$	4	$(x - 2)^2 = 0$ $x =$	(,)	
3.	$h(x) = -(x - 1)^2 - 3$				
4.	$y = -(x - 2)^2$				

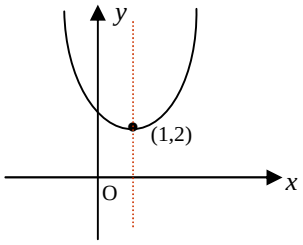
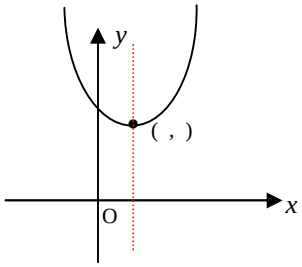
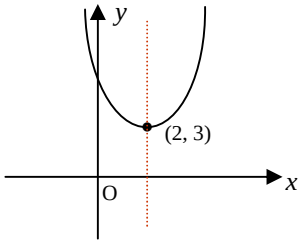
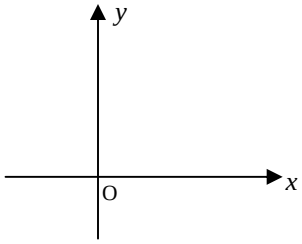
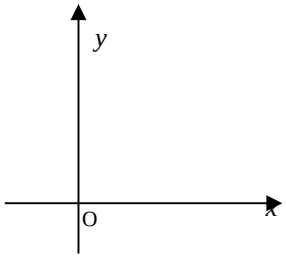
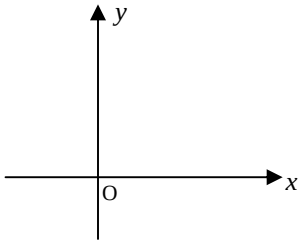
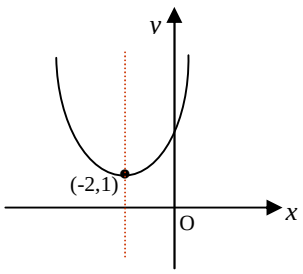
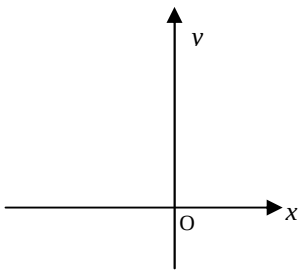
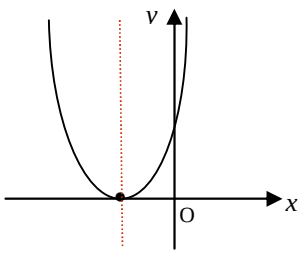
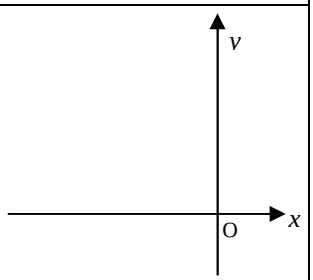
No.	Function	Minimum value of y	Corresponding value of x	Minimum Point	(Sketched) Graph
5.	$f(x) = -(x-3)^2 + 2$				
6.	$f(x) = -(x+2)^2 + 3$				
7.	$f(x) = -(x+1)^2 - 4$				
8.	$f(x) = -2(x+3)^2$				
9.	$f(x) = -2(x-1)^2 + 3$				
10.	$f(x) = -3(x+2)^2 - 1$				
11.	$f(x) = 2 - (x+1)^2$				
12.	$f(x) = 1 - 2(x-3)^2$				

3.2.6 To sketch the Graphs of Quadratic Functions in the form $f(x) = a(x + p)^2 + q$ and state the equation of the axis of symmetry.

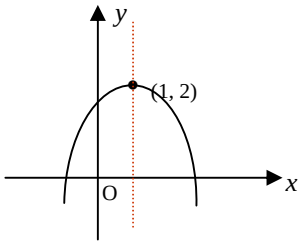
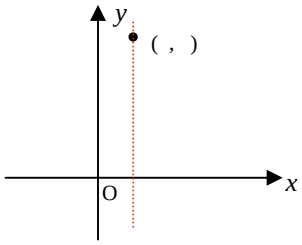
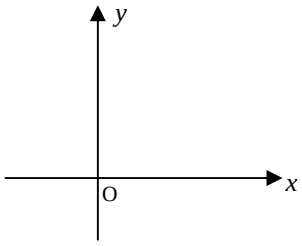
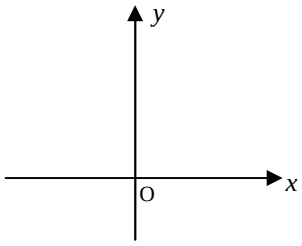
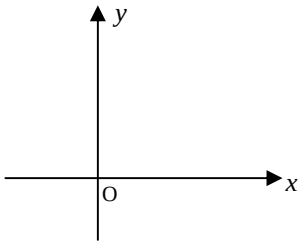
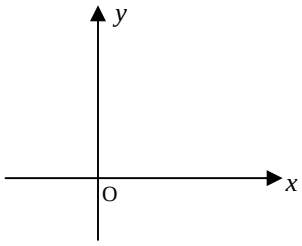
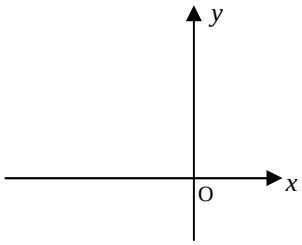
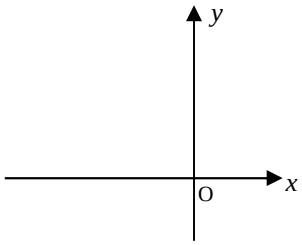
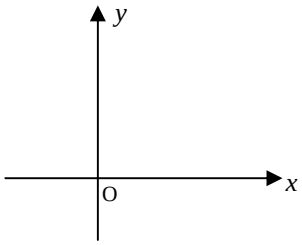
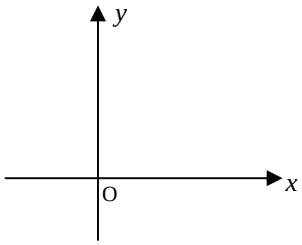
Note : The equation of the axis of symmetry is obtained by letting $(x + p) = 0$,

that is, $x = -p$

Case I : $a > 0$ $\Rightarrow$ **Shape of Graph is** ☺ **atau** ∪

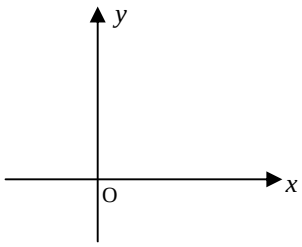
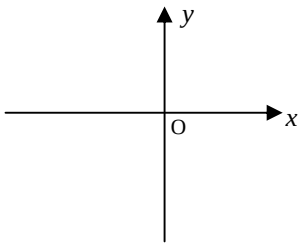
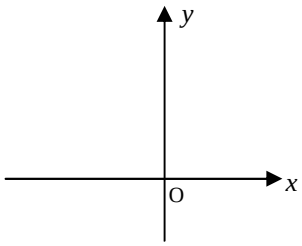
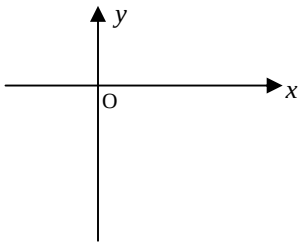
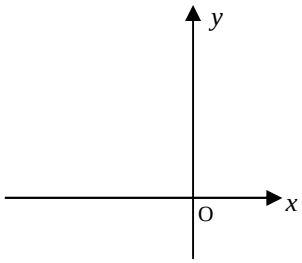
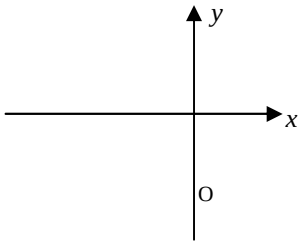
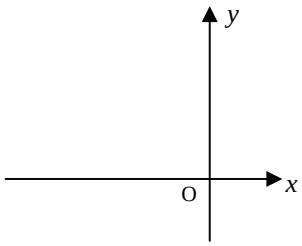
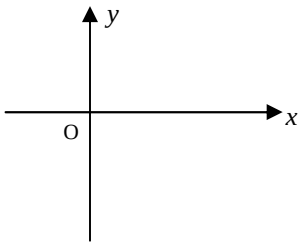
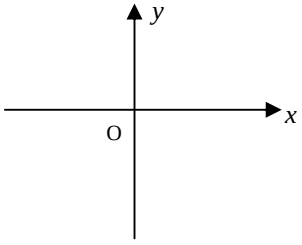
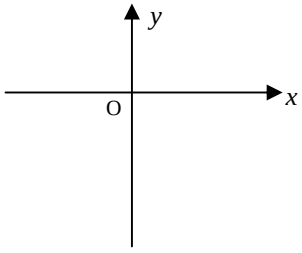
No.	Function	Sketched Graph	Function	Sketched Graph
1.	$f(x) = (x - 1)^2 + 2$ Min. Point : (1, 2) Axis of Symmetry : $x = 1$		$f(x) = (x - 1)^2 + 4$ Min. Point. : (,) Axis of Symmetry : $x =$	
2.	$f(x) = (x - 2)^2 + 3$ Min. Point. : (2, 3) Axis of Symmetry : $x = 2$		$f(x) = (x - 3)^2 + 2$ Min. Point : (,) Axis of Symmetry : $x =$	
3.	$f(x) = (x - 4)^2 + 2$ Min. Point. : (4,) Axis of Symmetry : $x =$		$f(x) = (x - 1)^2 + 3$ Min. Point : (,) Axis of Symmetry : $x =$	
4.	$f(x) = (x + 2)^2 + 1$ Min. Point. : (-2, 1) Axis of Symmetry : $x = -2$		$f(x) = (x + 1)^2 + 2$ Min. Point. : (, 2) Axis of Symmetry : $x =$	
5.	$f(x) = (x + 3)^2$ Min. Point. : (,) Axis of Symmetry : $x =$		$f(x) = (x + 4)^2$ Min. Point. : (,) Axis of Symmetry : $x =$	

Case 2 : a < 0 ⇒ **Shape of Graph :**  or 

1.	**f(x) = - (x – 1)^2 + 2** Max.Point : (1, 2) Axis of Symmetry : ***x* = 1**		**f(x) = - (x – 1)^2 + 4** Max.Point : (,) Axis of Symmetry : ***x* =**	
2.	**f(x) = - (x – 3)^2 + 1** Max.Point : (3, 1) Axis of Symmetry : ***x* =**		**f(x) = - x ^{2} + 2** Max. Point : (,) Axis of Symmetry : **=**	
3.	**f(x) = 3 - (x – 1)^2** Max.Point. : (, 3) Axis of Symmetry : ***x* =**		**f(x) = 5 - (x – 2)^2** Max.Point : (,) Axis of Symmetry :	
4.	**f(x) = - (x + 1)^2 + 4** Max.Point: (-1, 4) Axis of Symmetry : ***x* = -1**		**f(x) = - (x + 2)^2 + 2** Max.Point : (-2,) Axis of Symmetry : ***x* =**	
5.	**f(x) = - 2(x – 1)^2** Max.Point: (1,) Axis of Symmetry : ***x* =**		**f(x) = - (x – 3)^2** Max.Point : (,) Axis of Symmetry : ***x* =**	

GRAPHS OF QUADRATIC FUNCTIONS

3.2.7 Reinforcement exercises : To sketch graphs of Q.F. $f(x) = a(x+ p)^2 + q$

No.	Function	Sketched Graph	Function	Sketched Graph
1.	**$f(x) = (x – 2)^2 - 1$** Min. Point : (,) Axis of Symmetry : $x =$		**$f(x) = (x + 1)^2 - 4$** ……. Point : (,) Axis of Symmetry : $x =$	
2.	**$f(x) = 3 – 2 (x + 1)^2$** Max. point : (,) Axis of Symmetry : $x =$		**$f(x) = - 2 (x – 1)^2$** ……. Point : (,) Axis of Symmetry :	
3.	**$f(x) = (x + 1)^2 + 2$** ……. Point : (,) Axis of Symmetry : $x =$		**$f(x) = 1 – ½ (x + 2)^2$** ……. Point: (,) Axis of Symmetry :	
4	**$f(x) = (x + 3)2$** ……. Point**: (,)** **Axis of symmetry :** **x =**		**$f(x) = 9 - 4(x - 1)^2$** ……. Point: (,) Axis of Symmetry :	
5.	**$f(x) = x2 – 9$** ……. Point **: (,)** Axis of Symmetry : **x =**		**$f(x) = -3x^2 – 3$** ……. Point: (,) Axis of Symmetry :	

3.3.1 To express Quadratic Functions $f(x) = ax^2 + bx + c$ in the form $a(x+p)^2 + q$: Method of COMPLETING THE SQUARE

SIMPLE TYPE (a = 1)

	EXAMPLE	EXERCISE
1.	$f(x) = x^2 + 4x + 5$ $= x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 + 5$ $= (x+2)^2 - 4 + 5$ $= (x+2)^2 + 1$	$f(x) = x^2 + 4x + 3$ $(x+2)^2 - 1$
2.	$g(x) = x^2 - 6x + 8$ $= x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 8$ $= (x-3)^2 - 9 + 8$ $= (x-3)^2 - 1$	$g(x) = x^2 - 6x - 7$ $(x-3)^2 - 16$
3.	$h(x) = x^2 - 4x$ $= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2$ $= (x-2)^2 - 4$ $= (x-2)^2 - 4$	$h(x) = x^2 + 2x$ $(x+1) - 1$
4.	$y = x^2 - 4x + 5$ $= x^2 - 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 + 5$ $= (x-2)^2 - 4 + 5$ $= (x-2)^2 + 1$	$y = x^2 + x - 6$ $(x + ½)^2 - 25/4$
5.	$f(x) = x^2 + 5x + 6$ $= x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6$ $= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + 6$ $= \left(x + \frac{5}{2}\right)^2 - \frac{1}{4}$	$f(x) = x^2 + 3x + 2$ $(x + 3/2)^2 - ¼$

3.3.2 To express Q.F. $f(x) = ax^2 + bx + c$ in the form $a(x+p)^2 + q$: Method of COMPLETING THE SQUARE

When $a > 0$, $a \neq 1$.

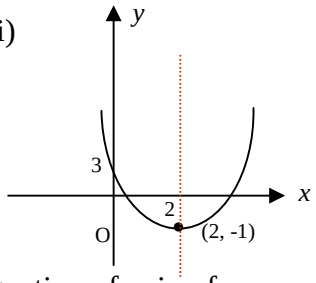
	EXAMPLE	EXERCISE
1.	$f(x) = 2x^2 + 4x + 6$ $= 2\left[x^2 + 2x + 3\right]$ $= 2\left[x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 3\right]$ $= 2\left[(x+1)^2 - 1 + 3\right]$ $= 2\left[(x+1)^2 + 2\right]$ $= \mathbf{2(x+1)^2 + 4}$	$f(x) = 2x^2 + 8x + 4$ $= 2\left[x^2 \quad\quad\right]$ $2(x+2)^2 - 4$
2.	$g(x) = 2x^2 + 6x - 5$ $= 2\left[x^2 + 3x - \frac{5}{2}\right]$ $= 2\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - \frac{5}{2}\right]$ $= 2\left[(x+\frac{3}{2})^2 \quad - \frac{9}{4} \quad - \frac{5}{2}\right]$ $= 2\left[(x+\frac{3}{2})^2 - \frac{19}{4}\right]$ $=$	$g(x) = 2x^2 - 6x + 3$ $2(x - 3/2)^2 - 3/2$
3.	$h(x) = 3x^2 + 6x - 12$ $= 3\left[x^2 + \quad\quad\right]$ $=$ $=$ $=$ $3(x+1)^2 - 15$	$g(x) = 3x^2 - 12x + 10$ $3(x-2)^2 - 2$

Questions based on SPM Format (1)

<table>
<tr><th></th><th>EXAMPLE</th><th>EXERCISE</th></tr>
<tr><td>C1</td><td>Express $f(x) = x^2 - 4x + 3$ in the form $(x + p)^2 + q$; with p and q as constants. Hence
(i) State the minimum value of f(x) and the corresponding value of x,
(ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry.

<u>Answers</u> : a = 1 (> 0) $\Rightarrow$ f has minimum value.
$f(x) = x^2 - 4x + 3$
$= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 3$
$= (x-2)^2 - 4 + 3$
$= (x-2)^2 - 1$

(i) Minimum value of f(x) = -1 , when x = 2.

(ii)
 

Equation of axix of symmetry : x = 2.</td><td>L1. Express $f(x) = x^2 - 6x + 8$ in the form $(x + p)^2 + q$; with p and q as constants. Hence
(i) State the minimum value of f(x) and the corresponding value of x,
(ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry.

<u>Ans</u> :

p = -3 , q = - 1</td></tr>
<tr><td>L2</td><td>Express $f(x) = x^2 + 2x - 3$ in the form $(x + p)^2 + q$. Hence
(i) State the minimum value of f(x) and the corresponding value of x.
(ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry.

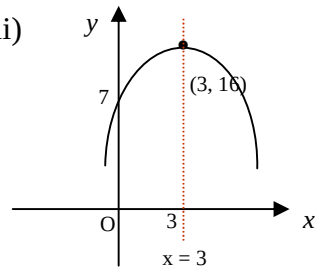
<u>Ans</u> :

p = 1 , q = -4</td><td>L3. Express $f(x) = x^2 + x + 2$ in the form $(x+ p)^2 + q$. Hence
(i) State the minimum value of f(x) and the corresponding value of x.
(iii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry.

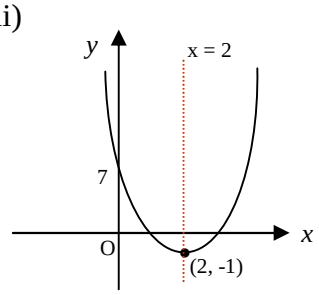
<u>Ans :</u>

p = ½ , q = 7/4</td></tr>
</table>

Questions based on SPM Format (II)

	EXAMPLE	EXERCISE
C2	Express $f(x) = -x^2 + 6x + 7$ in the form $k - (x + p)^2$, k and p are constants. Hence (i) State the maximum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. ***Ans: a = -1 (< 0) $\Rightarrow$ f has maximum value*** $f(x) = -x^2 + 6x + 7$ $= -\left[x^2 - 6x - 7\right]$ $= -\left[(x^2 - 6x + \left(\frac{-6}{3}\right)^2 - \left(\frac{-6}{2}\right)^2 - 7\right]$ $= -\left[\ (x-3)^2 - \ 9\ - 7\ \right]$ $= -\ [\ (x-3)^2 \ - \ 16\ ]$ $= 16 - (x-3)^2$ (i) Maximum f(x) = 16, when x = 3. (ii)  Axis of symmetry is : x = 3.	L4. Express $f(x) = -x^2 - 8x + 9$ in the form $-(x + p)^2 + q$. Hence (i) State the maximum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Ans:* **p = 4 , q = 25**
L5	Express $f(x) = -x^2 + 4x + 1$ in the form $-(x + p)^2$. Hence (i) State the maximum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Jawapan* : *(sila gunakan kertas sendiri)* $5 - (x - 2)^2$	L6. Express $f(x) = 4 - 3x - x^2$ in the form $q - (x + p)^2$ Hence (i) State the maximum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Jawapan* : $25/4 - (x + 3/2)^2$

Questions based on SPM Format (III)

	EXAMPLE	EXERCISE
C3	Express $f(x) = 2x^2 - 8x + 7$ in the form $a(x + p)^2 + q$, dengan a, p dan q pemalar. Seterusnya (i) State the minimum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. ***Ans:*** : a = 2 (> 0) $\Rightarrow$ f minimum value $f(x) = 2x^2 - 8x + 7$ $= 2(x^2 - 4x) + 7$ $= 2\left[x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2\right] + 7$ $= 2\left[(x-2)^2 - 4\right] + 7$ $= 2(x-2)^2 - 8 + 7$ $= 2(x-2)^2 - 1$ (i) Minimum value f(x) = -1 , when x = 2. (ii)  Axis of symmetry : **x = 2**.	L7. Express $f(x) = 2x^2 + 4x - 3$ in the form $a(x + p)^2 + q$. Seterusnya (i) State the minimum value of f(x) and state the coressponding value of x, (ii) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Ans :* **$2(x+1)^2 - 5$**
L8	Express $f(x) = 2x^2 + x - 6$ in the form $a(x + p)^2 + q$. Seterusnya (iii) State the minimum value of f(x) and state the coressponding value of x, (iv) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Jawapan* : *(sila gunakan kertas sendiri)* $2(x + 1/4)^2 - 49/8$	L9. Express $f(x) = 5 - 8x - 2x^2$ in the form $q - (x + p)^2$. Seterusnya (v) State the maximum value of f(x) and state the coressponding value of x, (vi) Sketch the graph of y = f(x) and state the equation of the axis of symmetry. *Jawapan* : $13 - 2(x+2)^2$

3.4 Quadratic Inequalities

(Students must be able to solve simple linear inequlities first)

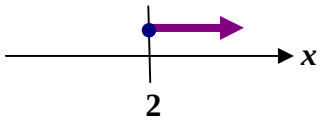
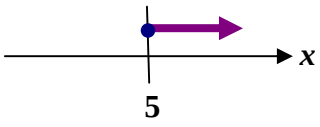
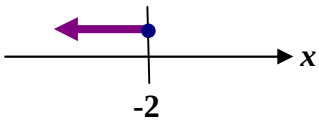
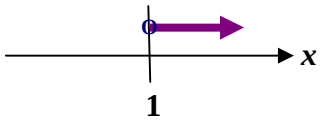
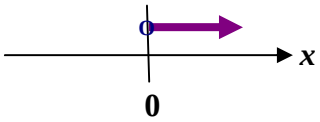
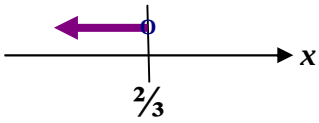
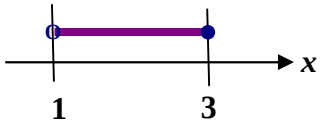
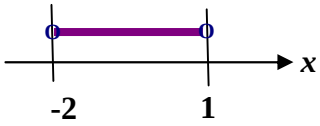
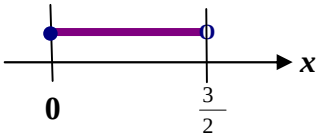
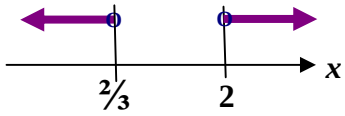
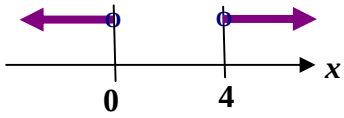
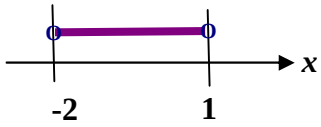
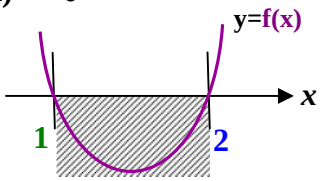
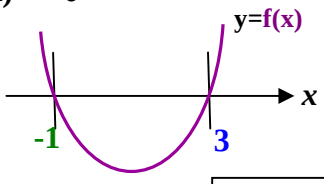
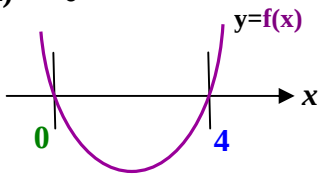
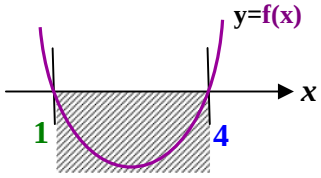
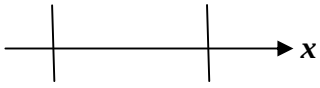
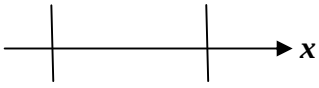
3.4.1 To Solve Simple Linear Inequalities (*Back to Basic*)

No.	EXAMPLE	EXERCISE 1	EXERCISE 2	EXERCISE 3
1.	$2x - 3 > 5$ $2x > 8$ $x > 4$	(a) $3x - 2 > 10$	(b) $3 + 4x < 21$	(c) $10 + 3x < 1$
2.	$-2x > 6$ $x < \frac{6}{-2}$ $x < -3$	(a) $-3x > 6$	(b) $-4x < -20$	(c) $-\frac{1}{2}x > 2$
3.	$3 - 4x > 9$ $-4x > 6$ $x < -\frac{6}{4}$ $x < -\frac{3}{2}$	(a) $3x - 2 > 10$	(b) $3 + 4x < 21$	(c) $10 + 3x < 1$
4.	$\frac{1-2x}{3} < 1$ $1 - 2x < 3$ $-2x < 2$ $x > \frac{2}{-2}$ $x > -1$	(a) $\frac{2-x}{4} > 2$	(b) $\frac{3-4x}{5} < 3$	$\frac{2-5x}{3} < 4$
5.	(a) $-\frac{x}{4} > 1$	(b) $-\frac{3-x}{2} < 4$	(c) $\frac{5x-3}{4} < -2$	(d) $\frac{x}{3} - 1 > 2x$

3.4.2 To Solve linear inequlities which involve two variables

No	EXAMPLE	EXERCISE 1	EXERCISE 2
1.	Given $2x + 3y = 10$. Find the range of x if $y > 4$. $2x + 3y = 10$ $3y = 10 - 2x$ $y = \frac{10 - 2x}{3}$ $\frac{10 - 2x}{3} > 4$ $10 - 2x > 12$ $-2x > 2$ $\boldsymbol{x < -1}$	(a) Given $2x - 3y = 12$. Find the range of x if $y > 2$. $x > 9$	(b) Given $4x - 3y = 15$. Find the range of x if $y < -3$. $x < 3/2$
2.	Given $x = \frac{3 - y}{3}$. Find the range of x if $y > 6$. $x = \frac{3 - y}{3}$ $3x = 3 - y$ $y = 3 - 3x$ $\therefore 3 - 3x > 6$ $-3x > 3$ $\boldsymbol{x < -1}$	(a) Given $x = \frac{5 - y}{3}$. Find the range of x if $y > 14$. $x < -3$	(b) Given $x = \frac{10 - 3y}{2}$. Find the range of x if $y \leq -2$. $x \geq 8$
3.	(a) Find the range of x if $2y - 1 = 3x$ and $4y > 12 + x$ **$x > 2$**	(b) Find the range of x if $6y - 1 = 3x$ and $3y > 2 + x$. $x > 3$	(c) Find the range of x if $2 - 3y = 4x$ and $y \leq 4$. $x \geq -5/2$
4	(a) Find the range of x if $3 + 2x > 5$ and $7 - 2x > 1$ $1 < x < 3$	(b) Find the range of x if $5 + 2x > 3$ and $9 - 2x > 1$ $-1 < x < 4$	(c) Find the range of x if $4 - 3x < 7$ and $-2x + 10 > 0$. $-1 < x < 5$

3.4.3 To state the range of values of x (with the help of a line graph)

No	EXAMPLE	EXERCISE 1	EXERCISE 2
1.	 **Inequality : $x \geq 2$** **(Range of values of x)**		
2.	 $x > 1$		
3.	 **Range of x : $1 < x \leq 3$**	 **Range of x :**	 **Range of x :**
4.	 **Range of x :** **$x < ⅔$ atau $x > 2$**	 **Range of x :**	 **Range of x :**
5.	**Given $f(x) = ax^2+bx+c$, $a>0$** **$f(x) < 0$**  **Range of x : $1 < x < 2$**	**Given $f(x) = ax^2+bx+c$, $a>0$** **$f(x) < 0$**  **Range of x :**	**Given $f(x) = ax^2+bx+c$, $a>0$** **$f(x) < 0$**  **Range of x :**
6.	**Solve $(x-1)(x-4) < 0$**  **Range of x : $1 < x < 4$**	**Solve $(x+2)(x-4) < 0$**  **Range of x :**	**Solve $x(x+3) < 0$**  **Range of x :**

3.4.5 Solving Quadratic Inequalities [*by sketching the graph of y = f(x)*]

Guide

STEP 1 : Make sure the inequality has been rearranged into the form f(x) **< 0** or f(x) **> 0**
(***Right-Hand Side MUST be 0 !***)

Example 1

$x^2 - 4x > 5$ changed to

$\mathbf{x^2 - 4x - 5 > 0}$

Example 2

$x(2x - 1) < 6$

$2x^2 - x < 6$

$\mathbf{2x^2 - x - 6 < 0}$

STEP 2 : Factorise f(x). [Here we consider only *f(x) which can be factorised*]
It is easier to factorise if ***a is made positive***.

Example

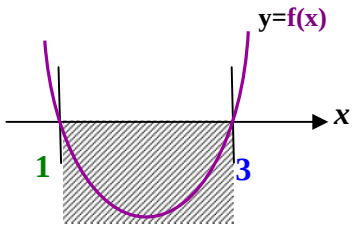
$-x^2 + 3x + 4 > 0$ can be transformed into

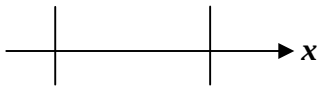
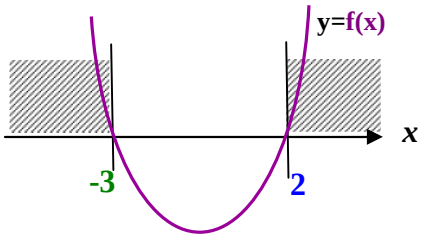
$\mathbf{x^2 - 3x - 4 < 0}$

Hence $\mathbf{(x+1)(x-4) < 0}$

STEP 3 : Sketch the graph of y = f(x) and shade the region which satisfy the inequality.

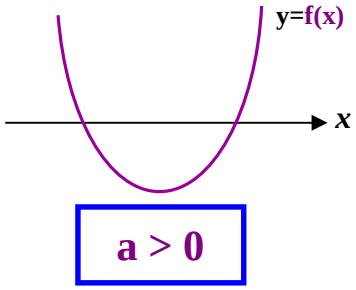
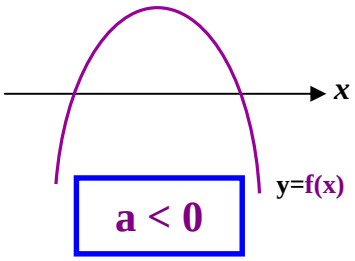
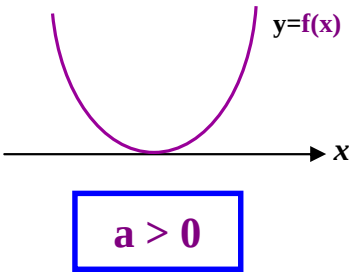
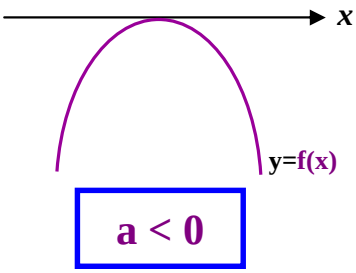
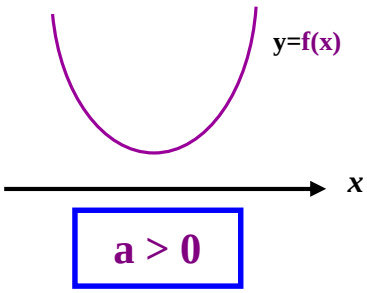
STEP 4 : State the range of values of x based on the graph.

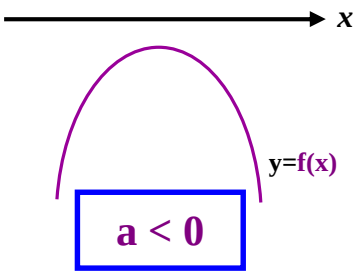
	EXAMPLE	**EXERCISE**
C1	**Solve $x^2 - 4x < -3$** $x^2 - 4x + 3 < 0$ [In the form f(x) < 0] $(x - 1)(x - 3) < 0$ [faktorise] Consider $f(x) = (x - 1)(x - 3)$ $f(x) = 0 \Rightarrow x = 1$ atau $x = 3$  **From the graph above, the range of x which satisfies the inequality f(x) < 0 ialah** **$1 < x < 3$**.	**L1. Solve $x^2 - 5x + 6 < 0$** **$2 < x < 3$**

	EXAMPLE	EXERCISE
L2	**Solve $x(x+4) < 12$** **$x(x+4) < 12$** **$x^2 + 4x - 12 < 0$** **[in the form f(x) = 0]** **( ) ( ) < 0** **[faktorise]** **Consider f(x) =** **f(x) = 0 $\Rightarrow$ x = or x =**  **From the graph above, the range of x which satisfies the inequality f(x) < 0 ialah**	**L3. Finf the range of values of x which satisfies $x^2 + 2x < 0$.** $-2 < x < 0$
C2	**Solve the inequality $x^2 + x - 6 \geq 0$** **$x^2 + x - 6 \geq 0$** **$(x + 3)(x - 2) \geq 0$** Consider f(x) = 0. Then x = -3 , x = 2  **Range of x is : $x \leq -3$ atau $x \geq 2$**	**L4. Solve the inequality $x^2 + 3x - 10 \geq 0$.** **$x \leq -5$, $x \geq 2$**
L5	**Solve the inequality $2x^2 + x > 6$.** $x < -2$, $x > 3/2$	**L6. Solve the inequality $x(4 - x) \geq 0$.** $0 \leq x \leq 4$

3.4.6 Quafratic Function $f(x) = ax^2 + bx + c$

Relationship between the value of "$b^2 - 4ac$" and the position of the graph $y = f(x)$

<table>
<tr><td>Case1</td><td colspan="2">$b^2 - 4ac > 0$
Graph $y = f(x)$ cuts x-axis at TWO different points.</td></tr>
<tr><td></td><td> 
</td><td> 
</td></tr>
<tr><td>Case 2</td><td colspan="2">$b^2 - 4ac = 0$
Graph $y = f(x)$ touches the x-axis.</td></tr>
<tr><td></td><td> 
</td><td> 
</td></tr>
<tr><td>Case 3</td><td colspan="2">$b^2 - 4ac < 0$
Graph $y = f(x)$ DOES NOT touch the x- axis.</td></tr>
<tr><td></td><td> 

Curve lies above the x-axis because f(x) is always positive.</td><td> 

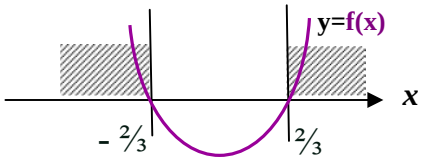
Curve lies below the x- axis because f(x) is always negative.</td></tr>
</table>

3.4.6 : Aplication (Relationship between "$b^2 - 4ac$" position of graph y = f(x)

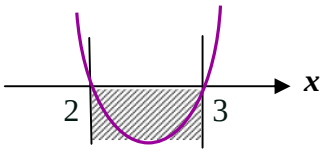
	EXAMPLE	EXERCISE
C1	**(SPM 2000)** **Show that the function $2x - 3 - x^2$ is always negative for all values of x.** **Ans : Let $f(x) = 2x - 3 - x^2$** **$= - x^2 + 2x - 3$** **$a = -1,\ b = 2,\ c = -3$** **$b^2 - 4ac = 2^2 - 4(-1)(-3)$** **$= 4 - 12$** **$< 0$** **Since $a < 0$ dan $b^2 - 4ac < 0$,** **the graph y = f(x) always lies above the x-axis** **$\therefore$ f(x) is always negative bagi semua x.** ***Note: The method of completing the squareshall be done later.***	**L1. Show that the function $4x - 2x^2 - 5$ is always negative for all values of x.**
L2	**Show that the function $2x^2 - 3x + 2x^2$ is always positive for all values of x.**	**L3. Show that the curve** **$y = 9 + 4x^2 - 12x$ touches the x-axis.**
C2	**Find the range of p if the graph of the quadratic function $f(x) = 2x^2 + x + 5 + p$ cuts the x-axis at TWO different points.** **Jawapan : $f(x) = 2x^2 + 6x + 5 + p$** **$a = 2,\ b = 1,\ c = 5 - p$** **$b^2 - 4ac > 0$** **$6^2 - 4(2)(5 + p) > 0$** **$36 - 40 - 8p > 0$** **$-8p > 4$** **$p < -\frac{1}{2}$**	**L4. Find the range of p if the graph of quarritic function $f(x) = x^2 + px - 2p$ cuts the x-axis at TWO different points.** **$p < -8,\ p > 0$**
L5	**The graph of the function $f(x) = 2x^2 + (3 - k)x + 8$ does not touch the x-axis. Determine the range of k.** **$-5 < k < 11$**	**L6. Find the values of k if the grapf of the quadratic function $y = x^2 + 2kx + k + 6$ touches the x-axis.** **$k = -3,\ k = 2$**

QUESTIONS BASED ON SPM FORMAT

<table>
<tr><th></th><th>EXAMPLE</th><th>EXERCISE</th></tr>
<tr><td>C1</td><td>(≈ SPM 1998)
(a) Given $f(x) = 9x^2 - 4$.
Find the range of x for which f(x) is positive.
(b) Find the range of x which satisfy the inequality $(x-2)^2 < x - 2$

Ans : (a) $f(x) > 0$
$9x^2 - 4 > 0$
$(3x + 2)(3x - 2) > 0$
$f(x) = 0 \Rightarrow x = -\frac{2}{3}, \frac{2}{3}$
 

$\therefore x < -\frac{2}{3}$ or $x > \frac{2}{3}$

(b) $(x-2)^2 < x - 2$
$x^2 - 4x + 4 - x + 2 < 0$
$x^2 - 5x + 6 < 0$
$(x-2)(x-3) < 0$
 

Range of x is $2 < x < 3$.</td><td>L1. (a) Given $f(x) = 2x^2 - 8$. Find the range of x so that f(x) is positive.
(b) Find the range of x which satisfy the inequality $(x-1)^2 > x - 1$

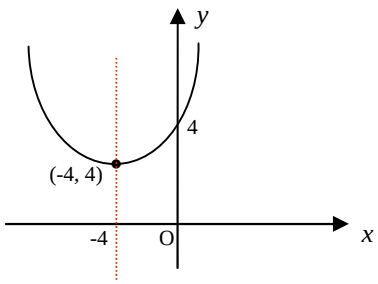
(Ans : (a) $x < -2, x > 2$ (b) $x < 1, x > 2$)</td></tr>
<tr><td>L2</td><td>(a) Find the range of x if $x(x + 2) \geq 15$
(b) State the range of x if $5x > 2 - 3x^2$.

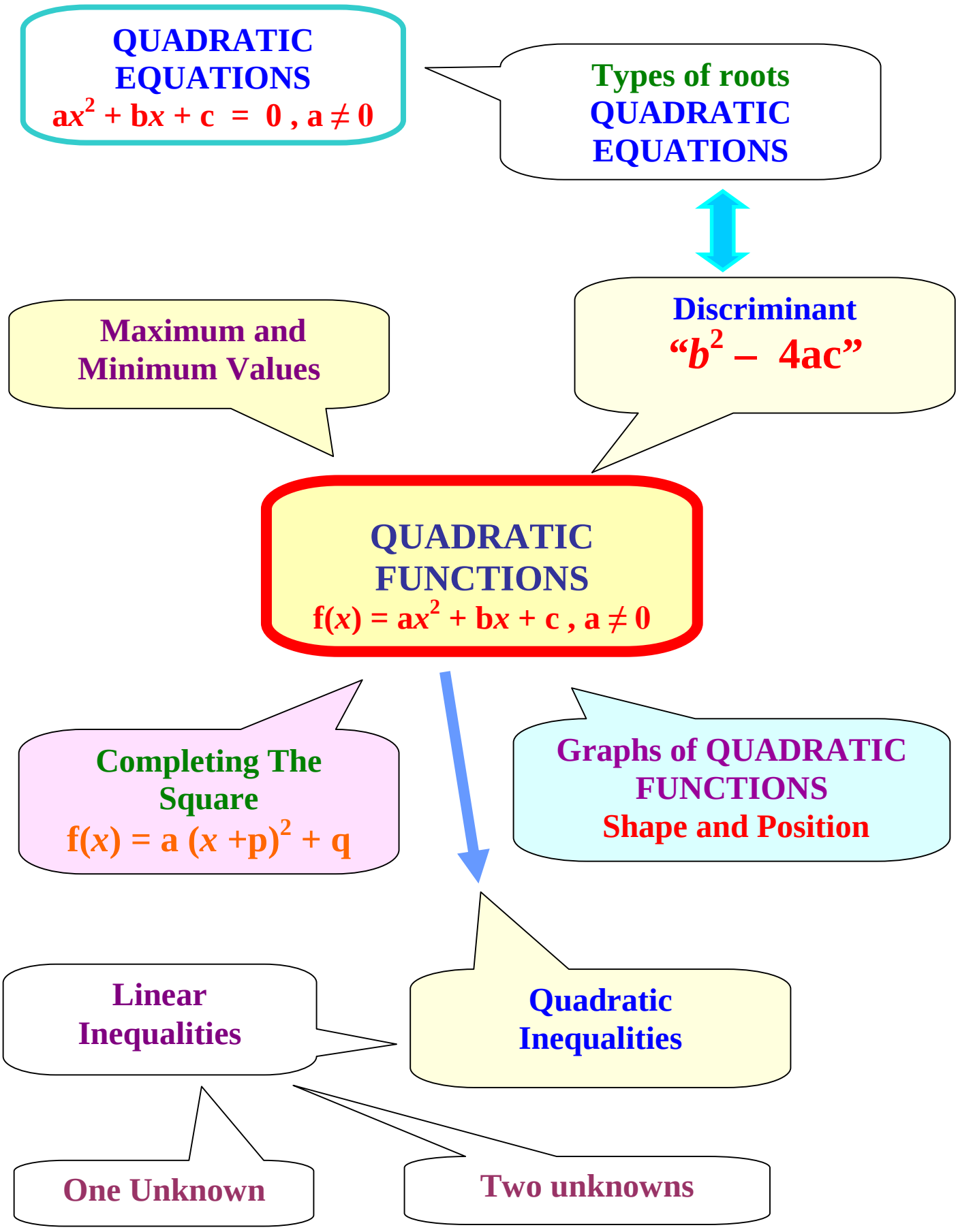
(a) $x \leq -5, x \geq 3$ (b) $x < -2, x > 1/3$</td><td>L3. (a) Solve $2x(x - 3) < 0$
(b) Find the values of x $x^2 > 4$.

(a) $0 < x < 3$ (b) $x < -2, x > 2$</td></tr>
<tr><td>L4</td><td>(a) Find the range of x if $3x(2x + 3) \geq 4x + 1$
(b) Solve $5 + m^2 > 9 - 3m$.

(a) $x < -1, x > 1/6$ (b) $m < -4, m > 1$</td><td>L5. (a) Solve $-2x(x + 3) > 0$
(b) Find the range of x if $9x^2 > 4$.

(a) $-3 < x < 0$ (b) $x < -2/3, x > 2/3$</td></tr>
</table>

	EXAMPLE /EXERCISE	EXERCISE
C2	Given $f(x) = x^2 + 2kx + 5k$ (k constant) has a minimum value 4. (a) By completing the square, determine the TWO positive values of k (b) Sketch the graph of y = f(x) for the bigger value of k and state the equation of the axis of symmetry. *Answer:* (a) $f(x) = x^2 + 2kx + 5k$ $= x^2 + 2kx + \left(\frac{2k}{2}\right)^2 - \left(\frac{2k}{2}\right)^2 + 5k$ $= (x + k)^2 - k^2 + 5k$ $\therefore -k^2 + 5k = \mathbf{4}$ (minimum value) $k^2 - 5k + 4 = 0$ $(k - 1)(k - 4) = 0$ $k = 1$ or $k = 4$ (b) $k = 4$, $f(x) = x^2 + 8x + 20$ $= x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 20$ $= (x + 4)^2 - 16 + 20$ $= (x + 4)^2 + 4$ (ii)  Axis of symmetry : x = - 4.	**L6**. Given $f(x) = x^2 + kx + 3$ (k constant) has a minimum value k. (a) By completing the square, determine the possible values of k (b) Sketch the graph of y = f(x) for the value of k which id negative and state the equation of the axis of symmetry. (Ans: k = -6 atau 2)
L7	Diven $y = h + 4kx - 2x^2 = q - 2(x + p)^2$ (a) Find p and q in terms of h and / or k. (b) If h = -10 and k = 3, (i) State the equation of the axis of symmetry, (ii) Sketch the graph of y = f(x) (Ans : p = -k , $q = 2k^2 + h$; paksi simetri : x = 3)	**L8**. Sketch the graphs of (a) $y = x^2 + 3$ (b) $y = 2(x - 3)^2 - 1$


QUADRATIC EQUATIONS
$ax^2 + bx + c = 0 , a \neq 0$
Types of roots QUADRATIC EQUATIONS
Discriminant
"$b^2 - 4ac$"
Maximum and Minimum Values
QUADRATIC FUNCTIONS
$f(x) = ax^2 + bx + c , a \neq 0$
Completing The Square
$f(x) = a(x + p)^2 + q$
Graphs of QUADRATIC FUNCTIONS
Shape and Position
Linear Inequalities
Quadratic Inequalities
One Unknown
Two unknowns