# 2. QUADRATIC EQUATIONS

**IMPORTANT NOTES :**

(i) The general form of a quadratic equation is $ax^2 + bx + c = 0$; a, b, c are constants and $a \neq 0$.

(ii) Characteristics of a quadratic equation:

(a) Involves only ONE variable,

(b) Has an equal sign " = " **and can be expressed in the form $ax^2 + bx + c = 0$**,

(c) The highest power of the variable is 2.

## 2.1 Recognising Quadratic Equations

EXAMPLES

| No | Quafratic Equations (Q.E.) | NON Q.E. | WHY? |
|---|---|---|---|
| 1. | $x^2 + 2x - 3 = 0$ | $2x - 3 = 0$ | No terms in $x^2$ ( a = 0) |
| 2. | $x^2 = ½$ | $x^2 + \frac{2}{x} = 0$ | Term $\frac{2}{x}$ |
| 3. | $4x = 3x^2$ | $x^3 - 2x^2 = 0$ | Term $x^3$ |
| 4. | $3x(x - 1) = 2$ | $x^2 - 3x^{-1} + 2 = 0$ | Term $x^{-1}$ |
| 5. | $p - 4x + 5x^2 = 0$, p constant | $x^2 - 2xy + y^2 = 0$ | Two variables |

Exercise : State whether the following are quadratic Equations. Give your reason for Non Q.E.

| No | Function | Q.F. | Non Q.F. | WHY? |
|---|---|---|---|---|
| 0. | $3x - 2 = 10 - x$ | | √ | No terms in $x^2$ |
| 1. | $x^2 = 10^2$ | | | |
| 2. | $12 - 3x^2 = 0$ | | | |
| 3. | $x^2 + x = 6$ | | | |
| 4. | $2x^2 + ½ x - 3 = 0$ | | | |
| 5. | $-\frac{6}{x} = x$ | | | |
| 6. | $0 = x(x - 2)$ | | | |
| 7. | $2x^2 + kx - 3 = 0$, k constant | | | |
| 8. | $(m-1)x^2 + 5x = 2m$ , m constant | | | |
| 9. | $3 - (p+1)x^2 = 0$ , p constant | | | |
| 10. | $p(x) = x^2 + 2hx + k+1$, h, k constants | | | |
| 11. | $f(x) = x^2 - 4$ | | | |
| 12. | $(k-1)x^2 - 3kx + 10 = 0$ , k constant | | | |

## 2.2 The ROOTs of a quadratic Equation (Q.E.)

Note : "ROOT" refers to a specific value which satisfies the Q.E.

Example : Given Q.E. $x^2 + 2x - 3 = 0$

By substitution, it is found that :

$x = 1 , 1^2 + 2(1) - 3 = 0$

Hence 1 is a root of the quadratic equation $x^2 + 2x - 3 = 0$.

But if $x = 2, \ 2^2 + 2(2) - 3 \neq 0,$

We say that 2 *is NOT a root* of the given quadratic Equation.

| | **EXAMPLE** | **EXERCISE** |
|---|---|---|
| **C1.** | Determine if -2 is a root of the equation $3x^2 + 2x - 7 = 0$.<br><br>**$x = -2, \ 3(-2)2 + 2(-2) - 7 = 12 - 4 - 7$**<br>**$\neq 0$**<br>**Hence - 2 is NOT a root of the given equation.** | **L1**. Determine if 3 is a root of the equation $2x^2 - x - 15 = 0$. |
| **L2.** | **L1**. Determine if **3 is a root of the equation $x^2 - 2x + 3 = 0$.** | **L3**. Determine if ½ **is a root of the equation $4x^2 + 2x - 2 = 0$.** |
| **C2.** | **If -2 is a root of the quadratic equation $x^2 - kx - 10 = 0$, find k.**<br><br>**$x = -2, \ (-2)2 - k(-2) - 10 = 0$**<br>**$-4 + 2k - 10 = 0$**<br>**$2k = 14$**<br>**$k = 7$** | **L4. If 3 is a root of the equation $x^2 - 2kx + 12 = 0$ , find k.** |
| **L5.** | **If -2 and p are roots of the quadratic equation $2x^2 + 3x + k = 0$, find the value of k and p.** | **L6. If -1 are roots of the quadratic equation $px^2 - 4x + 3p - 8 = 0$, find p.** |

Do you know that

**If the PRODUCT of two numbers is zero, then either one or both the numbers must be zero ?**

**If $x\,y = 0$ ,**
**Then $x = 0$ or $y = 0$**
**or $x = y = 0$ (both are zeroes)**

**Example : If $(x - 2)(x + 3) = 0$ ,**
**Then $x - 2 = 0$ or $x + 3 = 0$ ;**
**i.e. $x = 2$ or $x = -3$ .**

**2 and -3 are called the *roots* of the equation $(x-2)(x+3) = 0$.**

# 2.3.1 To Solve Quadratic Equations : $ax^2 + bx + c = 0$

## I. By Factorisation

- This method can only be used if the quadratic expression can be factorised completely.

| | EXAMPLE | EXERCISE |
|---|---|---|
| **C1.** | **Solve the quadratic equation $x^2 + 5x + 6 = 0$.**<br>*Answer*: $x^2 + 5x + 6 = 0$<br>$(x + 2)(x + 3) = 0$<br>$x + 2 = 0$ or $x + 3 = 0$<br>$x = -2$ or $x = -3$ | **L1. Solve $x^2 - 4x - 5 = 0$.**<br>*Jawapan*:<br>**Ans : – 1 , 5** |
| **C2.** | **Solve the quadratic equation $2x(x - 1) = 6$.**<br>*Ans*: $2x(x - 1) = 6$<br>$2x^2 - x - 6 = 0$<br>$(2x + 3)(x - 2) = 0$<br>$2x + 3 = 0$ or $x - 2 = 0$<br>$x = -\frac{3}{2}$ or $x = 2$ | **L2. Solve $x(1 + x) = 6$.**<br>**Ans : – 3 , 2** |
| **L3.** | **Solve $(x - 3)^2 = 1$.**<br>**Ans : 2, 4** | **L4. Solve $1 + 2x^2 = 5x + 4$.**<br>**Ans : 1, 3/2** |
| **L3.** | **Solve $(2x - 1)^2 = 2x - 1$ .**<br>**Ans : ½ , 1** | **L4. Solve $5x^2 - 45 = 0$.**<br>**Ans : – 3 , 3** |
| **L5.** | **Selesaikan $(x - 3)(x + 3) = 16$.**<br>**Ans : – 5 , 5** | **L6. Selesaikan $3 + x - 4x^2 = 0$.**<br>**Ans : – ¾ , 1** |
| **L7.** | **Solve $x(x + 2) = 24$.**<br>**Ans : – 6 , 4** | **L8. Solve $2(x^2 - 9) = 5x$.**<br>**Ans : – 2 , 9/2** |

# 2.3.2 To Solve Quadratic Equations : $ax^2 + bx + c = 0$

## II. By 'Completing the Square'

- To express $\mathbf{ax^2 + bx + c}$ **in the form** $\mathbf{a(x + p)^2 + q}$

## Simple Case : When a = 1

| | EXAMPLE | EXERCISE |
|---|---|---|
| C1. | **Solve $x^2 + 4x - 5 = 0$ by method of 'completing the square'.**<br><br>$x^2 + 4x - 5 = 0$<br>$x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5 = 0$<br>$(x + 2)^2 - 4 - 5 = 0$<br>$(x + 2)^2 - 9 = 0$<br>$(x + 2)^2 = 9$<br>$x + 2 = \pm 3$<br>$x = -2 \pm 3$<br>$x = -5$ or $x = 1$ | **L1. Solve $x^2 + 4x + 3 = 0$ by method of 'completing the square'.**<br><br>**(Ans : – 1 , – 3 )** |
| C2. | **Solve $x^2 - 6x + 3 = 0$ by method of 'completing the square'. Give your answer correct to 3 decimal places.**<br><br>$x^2 - 6x + 3 = 0$<br>$x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 + 3 = 0$<br>$(x - 3)^2 - 9 + 3 = 0$<br>$(x - 2)^2 - 6 = 0$<br>$(x + 2)^2 = 6$<br>$x + 2 = \pm \sqrt{6}$<br>$x = -2 \pm \sqrt{6}$<br>$x = -$ or $x =$ | **L2. Solve $x^2 - 8x + 5 = 0$ , give your answers correct to 4 significant figures.**<br><br>**Ans : 7.317, 0.6834** |
| **L3**. | **Solve $x^2 - 2x - 9 = 0$ by completing the square, give your answers correct to 4 significant figures.**<br><br>**Ans : – 2.212 , 4.162** | **L4. Solve $x^2 + 10x + 5 = 0$ , give your answers correct to 4 significant figures.**<br><br>**Ans : – 0.5279, – 9.472** |

# 2.3.3 To Solve Quadratic Equations : $ax^2 + bx + c = 0$

## II. Method of completing the square

- by expressing **$ax^2 + bx + c$ in the form $a(x + p)^2 + q$**

**[a = 1, but involving fractions when completing the square]**

| | EXAMPLE | EXERCISE |
|---|---|---|
| **C3.** | **Solve $x^2 - 3x - 2 = 0$ by method of 'completing the square'. Give your answer correct to 4 significant figures.**<br>$x^2 - 3x - 3 = 0$<br>$x^2 - 3x + \left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2 - 2 = 0$<br>$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 2 = 0$<br>$\left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$<br>$x - \frac{3}{2} = \pm\sqrt{\frac{17}{4}}$<br>$x = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$<br>**x = - 0.5616 atau x = 3.562** | **L5. Solve $x^2 + 5x - 4 = 0$. Give your answer correct to 4 significant figures.**<br>**(Ans : 0.7016, -5.702)** |
| **L6.** | **Solve $x^2 + x - 8 = 0$. Give your answer correct to 4 significant figures.**<br>**(Ans : 2.372, -3.372)** | **L7. Solve $x^2 + 7x + 1 = 0$ , Give your answer correct to 4 significant figures.**<br>**(Ans : -0.1459, -6.8541)** |
| **L8.** | **Solve x( x + 5) = 5. Give your answer correct to 4 significant figures.**<br>**(Ans : 0.8541, -5.854)** | **L9. Solve x(2 + x) = 10 Give your answer correct to 4 significant figures.**<br>**(Ans : 2.317 , -4.317 )** |

## 2.3.4 To Solve Quadratic Equations : $ax^2 + bx + c = 0$

**II. Method of completing the square**

- To express $\mathbf{ax^2 + bx + c}$ **in the form** $\mathbf{a(x + p)^2 + q}$

**If $a \neq 1$ : *Divide both sides by a* first before you proceed with the process of 'completing the square'.**

| | **EXAMPLE** | **EXERCISE** |
|---|---|---|
| **C4.** | **Solve $2x^2 - 8x + 7 = 0$ by completing the square.**<br><br>$2x^2 - 8x + 7 = 0$<br>$x^2 - 4x + \frac{7}{2} = 0$ [÷ 2 first ]<br>$x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + \frac{7}{2} = 0$<br>$(x - 2)^2 - 4 + \frac{7}{2} = 0$<br>$(x - 2)^2 = ½$<br>$x - 2 = \pm\sqrt{\frac{1}{2}}$<br>$x = 2 \pm\sqrt{\frac{1}{2}}$<br>= **2.707 atau 1.293** | **L10. Solve $2x^2 - 12x + 5 = 0$ correct to two decimal places.**<br><br>**(Ans : 5.55 , 0.45 )** |
| **C5.** | **Solve $-x^2 - 4x + 1 = 0$ by completing the square.**<br><br>$-x^2 - 4x + 1 = 0$ [divide by (-1)]<br>$x^2 + 4x - 1 = 0$<br><br>(Ans : 0.2361, -4.236 ) | **L11. Solve $-2x^2 + 10x + 9 = 0$ correct to two decimal places.**<br><br>**(Ans : -0.78 , 5.78 )** |
| **L12.** | **Solve $-x^2 - 7x + 3 = 0$ by completing the square.**<br><br>**(Ans : -7.405, 0.4051)** | **L13. Solve $x(3 - 2x) = -6$ correct to two decimal places.**<br><br>**(Ans : -1.14 , 2.64)** |

## 2.3.5 To Solve Quadratic Equations : $ax^2 + bx + c = 0$

### III. Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

| | EXAMPLE | EXERCISE |
|---|---|---|
| **C1.** | **Solve $2x^2 - 8x + 7 = 0$ by using formul. Give your answer correct to 4 significant figures.**<br>**a = 2, b = -8 , c = 7**<br>$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(7)}}{2(2)}$<br>$= \frac{8 \pm \sqrt{8}}{4}$<br>**= 2.707 atau 1.293** | **L1. By using formula, solve $2x^2 - 12x + 5 = 0$. Give your answer correct to 4 significant figures.**<br>**(Ans : 5.550, 0.4505)** |
| **C2.** | **Solve $2x(2 - 3x) = -5$ by using formula, qive your answer correct to two decimal places.**<br>**$2x(2 - 3x) = -5$**<br>**$4x - 6x^2 = -5$**<br>**$6x^2 - 4x - 5 = 0$**<br>**a = , b = , c =**<br>**x =**<br>**(Ans : 1.31 , -0.64)** | **L2. By using formula, solve $3 - x^2 = -3(4x - 3)$ correct to two decimal places.**<br>**(Ans: 0.52 , 11.48 )** |
| **L3.** | **Solve $x(2x - 1) = 2$ by using formula, give your answer correct to two decimal places.**<br>**(Ans : 1.28, -0.78)** | **L4. Solve the quadratic equation $2x(x - 4) = (1-x)(x+2)$. Give your answer correct to 4 significant figures.**<br>(SPM 2003)<br>(Ans : 2.591 , - 0.2573 ) |
| **L5.** | **Solve $x^2 - 4x = 2$ by using formula. Give your answer correct to 4 significant figures.**<br>**(Ans : 4.449 , -0.4495)** | **L6. Solve the quadratic equation $x(x - 4) = (3 - x)(x + 3)$. Give your answer correct to two decimal places.**<br>**(Ans : 3.35 , -1.35 )** |

# 2.4 To Form Quadratic Equations from Given Roots

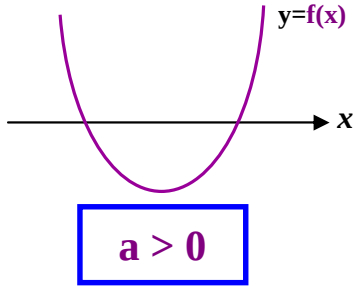
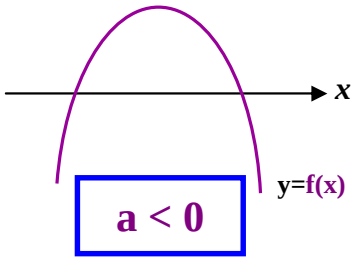
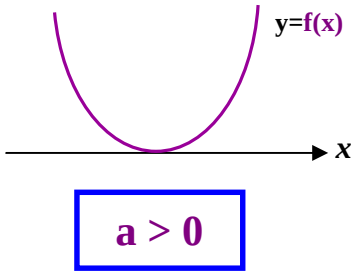
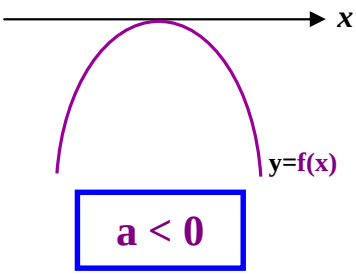
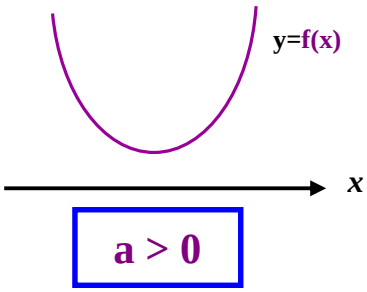
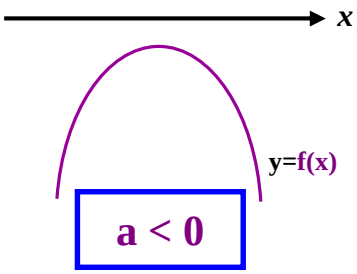
If *the roots* of a quadratic equation are $\alpha$ and $\beta$,
That is, $x = \alpha$ , $x = \beta$ ;
Then $x - \alpha = 0$ or $x - \beta = 0$ ,
$(x - \alpha)(x - \beta) = 0$
The quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

$x^2$ + Sum of Roots $x$ + Product of roots = 0

| | EXAMPLE | EXERCISE |
|---|---|---|
| C1. | **Find the quadratic equation with roots 2 dan - 4.**<br>$x = 2$ , $x = -4$<br>$x - 2 = 0$ or $x + 4 = 0$<br>$(x - 2)(x + 4) = 0$<br>$x^2 + 2x - 8 = 0$ | **L1. Find the quadratic equation with roots -3 dan 5.**<br>Ans : $x^2 - 2x - 15 = 0$ |
| L2. | **Find the quadratic equation with roots 0 dan - 3.**<br>$x = 0$ , $x = -3$<br>$x = 0$ or $x + 3 = 0$<br>Ans : $x^2 + 3x = 0$ | **L3. Find the quadratic equation with roots - ½ dan 6.**<br>$x = -\frac{1}{2}$ , $x = 6$<br>$2x = -1$ , $x = 6$<br>$2x + 1 = 0$ , $x - 6 = 0$<br>Ans : $2x^2 - 11x - 6 = 0$ |
| C2. | **Given that the roots of the quadratic equation $2x^2 + (p+1)x + q - 2 = 0$ are -3 and ½ . Find the value of p and q.**<br>$x = -3$ , $x = \frac{1}{2}$<br>$x + 3 = 0$ or $2x - 1 = 0$<br>$(x + 3)(2x - 1) = 0$<br>$2x^2 + 5x - 3 = 0$<br>Comparing with the original equation :<br>$p + 1 = 5$ , $q - 2 = -3$<br>$p =$ , $q =$ | **L4. Given that the roots of the quadratic equation $3x^2 + kx + p - 2 = 0$ are 4 and - ⅔. Find k and p.**<br>(Ans : k = -10 , p = -6) |
| L5. | **Given that the roots of the quadratic equation $x^2 + (h - 2)x + 2k = 0$ are 4 and -2 . Find h and k.**<br>(Ans : h = 0, k = -4) | **L6. Given that the roots of the quadratic equation $2x^2 + (3 - k)x + 8p = 0$ are p and 2p , $p \neq 0$. Find k and p.**<br>(Ans: p = 2, k = 15) |

## 2.5 The Quadratic Equation $ax^2 + bx + c = 0$

### 2.5.1 Relationship between "$b^2 - 4ac$" and the *Roots of the Q.E.*

<table>
<tr><td>CASE1</td><td colspan="2">$b^2 - 4ac > 0$<br>Q.E. has two distinct roots.<br>The Graph y = f(x) cuts the x-axis at TWO distinct points.</td></tr>
<tr><td></td><td> 
</td><td> 
</td></tr>
<tr><td>CASE 2</td><td colspan="2">$b^2 - 4ac = 0$<br>Q.E. has real and equal roots.<br>The graph y = f(x) touches the x-axis [ The x-axis is the tangent to the curve]</td></tr>
<tr><td></td><td> 
</td><td> 
</td></tr>
<tr><td>CASE 3</td><td colspan="2">$b^2 - 4ac < 0$<br>Q.E. does not have real roots.<br>Graph y = f(x) does not touch x-axis.</td></tr>
<tr><td></td><td> 
<br>Graph is above the x-axis since f(x) is always positive.</td><td> 
<br>Graph is below the x-axis since f(x) is always negative.</td></tr>
</table>

## 2.5.2 Aplication (Relationship between "$b^2 - 4ac$" and *the type of roots*)

| | EXAMPLE | EXERCISE |
|---|---|---|
| **C1** | **(SPM 2000)**<br>**The roots of the quadratic equation $2x^2 + px + q = 0$ are - 6 and 3.**<br>**Find**<br>**(a) p and q,**<br>**(b) range of values of k such that $2x^2 + px + q = k$ does not have real roots.**<br><br>**Answer :**<br>**(a)** $x = -6 \ , \ x = 3$<br>$(x + 6)(x - 3) = 0$<br>$x^2 + 3x - 18 = 0$<br>$2x^2 + 6x - 36 = 0$<br>**Comparing :** $p = 6 \ , \ q = -36.$<br><br>**(b)** $2x^2 + 6x - 36 = k$<br>$2x^2 + 6x - 36 - k = 0$<br>$a = 2, \ b = 6, \ c = -36 - k$<br>$b^2 - 4ac < 0$<br>$6^2 - 4(2)(-36 - k) < 0$<br>$324 + 8k < 0$<br>$k < -40.5$ | **L1. The roots of the quadratic equation $2x^2 + px + q = 0$ are 2 and -3.**<br>**Find**<br>**(a) p and q,**<br>**(b) the range of values of k such that $2x^2 + px + q = k$ does not have real roots.** |
| **L2** | **Find the range of k if the quadratic equation $2x^2 - x = k$ has real and distinct roots.**<br><br>**( Ans : $k > -1/8$ )** | **L3. The quadratic equation $9 + 4x^2 = px$ has equal roots. Find the possible values of p.**<br><br>**( Ans : p = -12 atau 12)** |
| **L4** | **Find the range of p if the quadratic equation $2x^2 + 4x + 5 + p = 0$ has real roots.**<br><br>**(Ans : $p \leq -3$ )** | **L5. Find the range of p if the quadratic equation $x^2 + px = 2p$ does not have real roots.**<br><br>**( Ans : $-8 < p < 0$ )** |
| **L6** | **The roots of the quadratic equation $2x^2 + 8 = (k - 3)x$ are real and different. Determine the range of values of k.**<br><br>**( Ans : $k < -5$ , $k > 11$ )** | **L7. Find the range of values of k if the quadratic equation $x^2 + 2kx + k + 6 = 0$ has equal roots.**<br><br>**( Ans : k -2 , 3 )** |

# Reinforcement Exercises (SPM Format Questions)

| | EXERCISE | EXERCISE |
|---|---|---|
| **L1** | (a) The equation $x^2 - 6x + 7 = h(2x - 3)$ has roots which are equal. Find the values of h. [4]<br>(b) Given that $\alpha$ and $\beta$ are roots of the equation $x^2 - 2x + k = 0$, while $2\alpha$ and $2\beta$ are the roots of the equation $x^2 + mx + 9 = 0$. Determine the possible values of k and m. [SPM 1999] [6]<br><br>( h = -1 , -2 ; k = $\frac{9}{4}$ | **L2. One of the roots of the equation $2x^2 + 6x = 2k - 1$ is twice the other. Find the value of k and the roots of the equation.** **[1999]**<br><br>**( x = -1 , x = -2 ; k = $-\frac{3}{2}$ )** |
| **L2**. | (SPM 2003 , P1, S3). Solve the quadratic equation $2x(x - 4) = (1 - x)(x + 2)$. Give your answer correct to 4 significant sigures. [3]<br><br>( x = 2.591, - 0.2573) | **L3.** (SPM 2003, P1, S4) The quadratic equation $x(x+1) = px - 4$ has two distinct roots.. Find the range of values of p. [3]<br><br>( p , -3 , p > 5) |
| **L4** | (SPM 2002) Find the range of k if the Q.E. $x^2 + 3 = k(x - 1)$, k constant, has two distinct roots. [3]<br><br>( k < -2 , k > 6) | **L5**. (≈ SPM 2001) Show that the straight line $y = 2 - x$ does not meet the curve $2x^2 - y^2 + k = 0$ if **k > 8**. [3] |

| | EXERCISE | EXERCISE |
|---|---|---|
| **L6** | (SPM 2002) Given $\frac{p}{2}$ and $\frac{q}{2}$ are roots of the equation $kx(x-1) = m - 4x$. If $p + q = 4$ and $pq = -5$, find the values of k and m [5]<br><br>( k = -4 , m = -5 ) P.S. *quite challenging!* | **L7**. (SPM 2001) Given 2 and m are roots of the equation $(2x - 1)(x + 3) = k(x - 1)$, with k as a constant, find k and m. [4]<br><br>( k = 15 , m = 3 ) |
| **L8.** | (SPM 2000) Find the range of x if the straight line $y = 2x + k$ does not intersect the curve $x^2 + y^2 - 6 = 0$. [5]<br><br>(k < -5.477 atau k > 5.477) | **L9**. (SPM 2000) The quadratic equation $2x^2 + px + q = 0$ has roots -2 and 3. Find **the value of p anf q so that** $2x^2 + px + q = k$ has real roots.<br><br>( p = -2 , q = -12 ; k ≥ - 12.5 ) |
| L10. | (SPM 1995)<br>(c) Given ½ and -5 are roots of a quadratic Equation.Write down the quadratic equation in the form $ax^2 + bx + c = 0$. [2]<br>(b) Find the range of values of x for which the equation $x^2 + kx + 2k - 3 = 0$ has no real roots. [3]<br><br>( $2x^2 + 9x - 5 = 0$ ; $2 < k < 6$ ) | (c) Prove that the roots of the equation $(1 - p)x^2 + x + p = 0$ are real and negative IF ***0 < p < 1***. [5] |

*U*ntuk renungan : Gred MT anda adalah berkadar songsang dengan latihan yang anda buat !