1.1.3 Terms in arithmetic progressions

(a) State the common difference of each of the following arithmetic progressions

1. 2, 6, 10, 14, … [4]	2. 21, 18, 15, 12, … [3]
3. -6, 1, 8, 15, … [7]	4. -12, -9, -6, -3, … [3]
5. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12}, \ldots$ [$\frac{1}{4}$]	6. $-\frac{1}{3}, -\frac{5}{6}, -\frac{3}{4}, -\frac{5}{4}, \ldots$ [$-\frac{1}{2}$]

(b) Find the tenth term and the twentieth term of the following arithmetic progressions

1. 2, 6, 10, 14, … [38 ; 78]	2. 21, 18, 15, 12, … [-6 ; -36]
3. -6, 1, 8, 15, … [57 ; 127]	4. -12, -9, -6, -3, … [15 ; 45]

5. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12}, \ldots$ $[\frac{31}{12}; \frac{61}{12}]$	6. $-\frac{1}{3}, -\frac{5}{6}, -\frac{3}{4}, -\frac{5}{4}, \ldots$ $[-\frac{29}{6}; -\frac{59}{6}]$

(c) Calculate the number of terms in each of the following arithmetic progressions

1. 2, 6, 10, …, 82 [21]	2. 21, 18, 15, …, -66. [30]
3. -6, 1, 8, …, 190 [29]	4. -12, -9, -6, …, 36 [17]
5. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, .., \frac{22}{3}$ [29]	6. $-\frac{1}{3}, -\frac{5}{6}, -\frac{3}{4}, ..., -\frac{46}{3}$ [31]

(d) Further Practice

1. The first three terms of an arithmetic progression are k-2, k+2, 2k+1. Find the value of k. [5]	2. The first three terms of an arithmetic progression are k-3, 2k-3, k+1. Find the value of k. [2]
3. The nth term of an arithmetic progression is given by $T_n = 3n + 1$. Find (a) the first term, (b) the common difference. [4 ; 3]	4. The nth term of an arithmetic progression is given by $T_n = 4n - 9$. Find (a) the first term, (b) the common difference. [-5 ; 6]
5. Given an arithmetic progression 2, 6, 10, 14, …, find the smallest value of n such that the nth term is greater than 100. [26]	6. Given an arithmetic progression -12, -9, -6, -3, … find the greatest value of n such that the nth term is smaller than 200. [71]
7. The third term and eighth term of an arithmetic progression are 6 and 31 respectively. Find the first term and the common difference. [-4 ; 5]	8. The fourth term and ninth term of an arithmetic progression are 9 and 29 respectively. Find the first term and the common difference. [-3 ; 4]

1.1.4 Finding the sum of an arithmetic progressions

(a) Find the sum of the first 20 terms of each of the following arithmetic progressions

1. 2, 6, 10, 14, … [800]	2. 21, 18, 15, 12, … [-150]
3. -6, 1, 8, 15, … [1210]	4. -12, -9, -6, -3, … [330]
5. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12}, \ldots$ [$\frac{325}{6}$]	6. $-\frac{1}{3}, -\frac{5}{6}, -\frac{3}{4}, -\frac{5}{4}, \ldots$ [$-\frac{305}{3}$]

(b) Find the sum of the following arithmetic progressions

1. 2, 6, 10, 14, …, 54 [1456]	2. 21, 18, 15, 12, …, -30 [-81]

3. -6, 1, 8, 15, …, 127 [1210]	4. -12, -9, -6, -3, …, 30 [135]

(c) Sum of a specific number of consecutive terms

1. Given an arithmetic progression 2, 6, 10, 14, … find the sum from fifth term to the sixteenth term. [504]	2. Given an arithmetic progression 21, 18, 15, 12, … find the sum from seventh term to the eighteenth term. [-180]
3. Given an arithmetic progression -6, 1, 8, 15, … find the sum from ninth term to the twentieth term. [1104]	4. Given an arithmetic progression -12, -9, -6, -3, find the sum from eleventh term to the twenty-eighth term. [810]

(d) Further Practice

1. Given an arithmetic progression 2, 6, 10, 14, …, find the value of n for which the sum of the first n terms is 800. [20]	2. Given an arithmetic progression 21, 18, 15, 12, … find the value of n for which the sum of the first n terms is -81. [18]
3. The sum of the first n terms of an arithmetic progression is given by $S_n = 2n^2 + n$. Find (a) the first term, (b) the common difference. [3 ; 4]	4. The sum of the first n terms of an arithmetic progression is given by $S_n = 2n^2 - 5n$. Find (a) the first term, (b) the common difference. [-3; 4]
5. Given an arithmetic progression 2, 6, 10, 14, … find the smallest value of n such that the sum of the first n terms is greater than 200. [11]	6. Given an arithmetic progression -12, -9, -6, -3, … find the smallest value of n such that the sum of the first n terms is greater than 243. [19]

7. The first and last terms of an arithmetic are 3 and 21 respectively and the sum of the series is 240. Find the number of terms. [20]	8. The first and last terms of an arithmetic progression are -4 and 18 respectively and the sum of the series is 168. Find the number of terms. [24]
9. The sum of the first four terms of an arithmetic progression is 36 and the sum of the next ten terms is 370. Find the first term and the common difference. [3 ; 4]	10. The sum of the first six terms of an arithmetic progression is 42 and the sum of the next twelve terms is 558. Find the first term and the common difference. [-3 ; 4]
11. The sixth term of an arithmetic progression is 23 and the sum of the first six terms is 78. Find the first term and the common difference. [3 ; 4]	12. The eighth term of an arithmetic progression is 25 and the sum of the first eight terms is 88. Find the first term and the common difference. [-3 ; 4]

1.2.3 Terms in geometric progressions

(a) State the common ratio of each of the following geometric progressions

1. 2, 6, 18, 54, … [3]	2. 3, 12, 48, 192, … [4]
3. 1458, 486, 162, 54, … $[\frac{1}{3}]$	4. 192, -96, 48, -24, … $[-\frac{1}{2}]$

(b) Find the fifth term and the tenth term of the following geometric progressions

1. 2, 6, 18, 54, … [162 ; 39366]	2. 3, 12, 48, 192, … [768 ; 786432]
3. 1458, 486, 162, 54, … $[18 ; \frac{2}{27}]$	4. 192, -96, 48, -24, … $[12 ; -\frac{3}{8}]$

(c) Calculate the number of terms in each of the following geometric progressions

1. 2, 6, 18, …, 1458 [6]	2. 3, 12, 48, …, 3072. [6]
3. 1458, 486, 162, …, 2 [7]	4. 192, -96, 48, … , $\frac{1}{64}$ [7]

(d) Further Practice

1. The first three terms of a geometric progression are k, k + 3, k + 9. Find the value of k. [3]	2. The first three terms of a geometric progression are k-1, k + 2, k + 8. Find the value of k. [4]

3. The nth term of a geometric progression is given by $T_n = 2^{2n-1}$. Find (a) the first term, (b) the common ratio. [2 ; 3]	4. The nth term of a geometric progression is given by $T_n = 3^{3n-2}$. Find (a) the first term, (b) the common ratio. [3 ; 27]
5. Given a geometric progression 2, 6, 18, 54, … find the smallest value of n such that the nth term is greater than 100000. [13]	5. Given a geometric progression 3, 12, 48, 192, … find the smallest value of n such that the nth term is greater than 180000. [9]
7. The second term and fifth term of a geometric progression are 12 and 96 respectively. Find the first term and the common difference. [± 3 ; ± 2]	8. The third term and sixth term of a geometric progression are 108 and 2916 respectively. Find the first term and the common difference. [3 ; 12]

1.2.4 Finding the sum of a geometric progressions

(a) Find the sum of the first 8 terms of each of the following geometric progressions

1. 2, 6, 18, 54, … [6560]	2. 3, 12, 48, 192, … [5535]
3. 1458, 486, 162, 54, … $[\frac{6560}{2187}]$	4. 192, -96, 48, -24, … [127.5]

(b) Find the sum of the following geometric progressions

1. 2, 6, 18, …, 4374 [2186]	2. 3, 12, 48, …, 3072. [4095]

3. 1458, 486, 162, …, 2 [2186]	4. 192, -96, 48, … , $\frac{3}{4}$ [129]

(c) Sum of a specific number of consecutive terms

1. Given a geometric progression 2, 6, 18, 54, … find the sum from fifth term to the ninth term. [19602]	2. Given a geometric progression 3, 12, 48, 192, … find the sum from sixth term to the tenth term. [1047552]
3. Given a geometric progression 1458, 486, 162, 54, …find the sum from fifth term to the eighth term. [26.67]	4. Given an arithmetic progression 192, -96, 48, -24,… find the sum from seventh term to the tenth term. [1.875]

(d) (i) Find the sum to infinity of each of the following geometric progressions

1. 8, 4, 2, 1, … [16]	2. 27, 9, 3, 1, … [40.5]
3. 1458, 486, 162, 54, … [2187]	4. 192, -96, 48, -24, … [128]

(d) (ii)

1. The first term and the sum to infinity of a geometric progression are 4 and 8 respectively. Find the common ratio. [$\frac{1}{2}$]	2. The first term and the sum to infinity of a geometric progression are 5and 15 respectively. Find the common ratio. [$\frac{2}{3}$]
3. The common ratio and the sum of a geometric progression are $\frac{1}{3}$ and 30 respectively. Find the first term. [20]	4. The common ratio and the sum of a geometric progression are $-\frac{1}{4}$ and 20 respectively. Find the first term. [25]

(d) (iii) Express each of the following recurring decimals as a fraction or mixed number in its simplest form

1. 0.5555… [$\frac{5}{9}$]	2. 0.7777… [$\frac{7}{9}$]
3. 0.060606… [$\frac{2}{33}$]	4. 0.020202… [$\frac{2}{99}$]
5. 0.0363636… [$\frac{2}{55}$]	6. 0.345345345… [$\frac{115}{333}$]
7. $3.0\dot{1}\dot{8}$ [$3\frac{1}{55}$]	8. $5.0\dot{1}\dot{2}$ [$5\frac{4}{33}$]

(e) Further Practice

1. Given a geometric progression 2, 6, 18, 54, … find the value of n for which the sum of the first n terms is 19682. [9]	2. Given a geometric progression 3, 12, 48, 192, … find the value of n for which the sum of the first n terms is 65535. [8]
3. The sum of the first n terms of a geometric progression is given by $S_n = 4^n - 1$. Find (a) the first term, (b) the common ratio. [a= 3 r = 4]	4. The sum of the first n terms of a geometric progression is given by $S_n = 1- (-3)^n$. Find (a) the first term, (b) the common ratio. [a = 4 r = -3]
5. Given a geometric progression 2, 6, 18, 54, … find the smallest value of n such that the sum of the first n terms is greater than 6000. [n = 8]	6. Given a geometric progression 3, 12, 48, 192, … find the smallest value of n such that the sum of the first n terms is greater than 2000. [n = 6]