

To find unknown sides : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

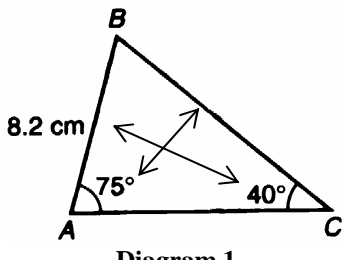
To find unknown angles : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
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SOLUTION OF TRIANGLES

1.2 Use Sine Rule to find the unknown sides or angles of a triangle.

Task 1 : Find the unknown sides of a triangle when two of its angles and one of the corresponding sides are known.

(1) Diagram 1 shows the triangle ABC. Answer :



$$\frac{BC}{\sin 75^{\circ}} = \frac{8.2}{\sin 40^{\circ}}$$

$$BC = \frac{8.2}{\sin 40^{\circ}} \times \sin 75^{\circ}$$

Diagram 1

Using the scientific calculator,

$BC = 12.32 \text{ cm}$

Calculate the length of BC.

(2) Diagram 2 shows the triangle PQR

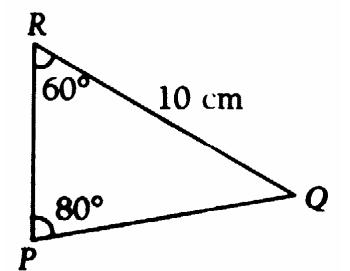


Diagram 2

Calculate the length of PQ. [8.794 cm]

(3) Diagram 3 shows the triangle DEF.

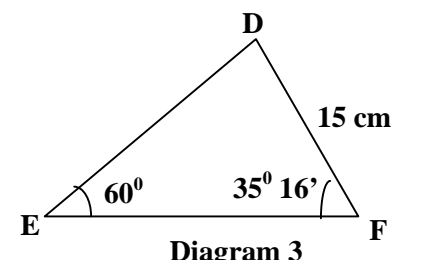


Diagram 3

Calculate the length of DE. [10.00 cm]

(4) Diagram 4 shows the triangle KLM.

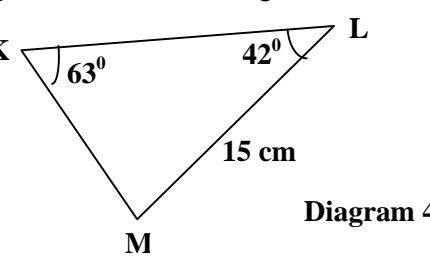


Diagram 4

Calculate the length of KM. [11.26 cm]

(5) Diagram 5 shows the triangle ABC.

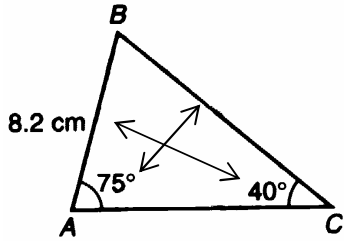


Diagram 5

Calculate the length of AC.

Answer :

$$\angle ABC = 180^\circ - 40^\circ - 75^\circ = 65^\circ$$

$$\frac{AC}{\sin 65^\circ} = \frac{8.2}{\sin 40^\circ}$$

$$AC = \frac{8.2}{\sin 40^\circ} \times \sin 65^\circ$$

Using the scientific calculator,

$$AC = 11.56 \text{ cm}$$

(6) Diagram 6 shows the triangle PQR

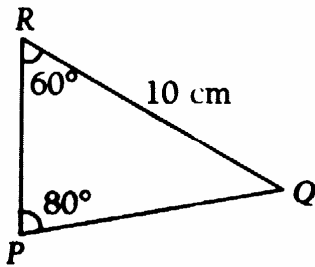


Diagram 6

Calculate the length of PR.

[6.527 cm]

(7) Diagram 7 shows the triangle DEF.

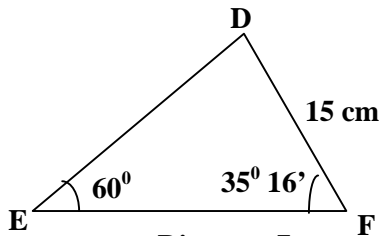


Diagram 7

Calculate the length of EF.

[17.25 cm]

(8) Diagram 8 shows the triangle KLM.

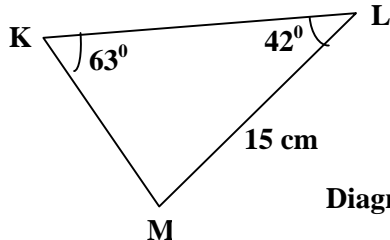


Diagram 8

Calculate the length of KL.

[16.26 cm]

Task 2 : Find the unknown sides of a triangle when two of its angles and the side not corresponding to the angles are known.

(9) Diagram 9 shows the triangle ABC.

Answer :

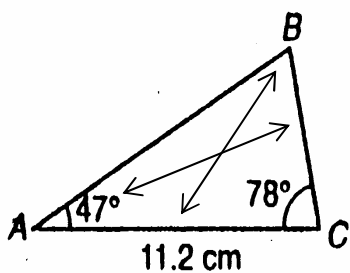


Diagram 9

$$\angle ABC = 180^\circ - 47^\circ - 78^\circ = 55^\circ$$

$$\frac{BC}{\sin 47^\circ} = \frac{11.2}{\sin 55^\circ}$$

$$BC = \frac{11.2}{\sin 55^\circ} \times \sin 47^\circ$$

Calculate the length of BC.

Using scientific calculator,

$$BC = 9.9996 \text{ cm or } 10.00 \text{ cm}$$

(10) Diagram 10 shows the triangle ABC.

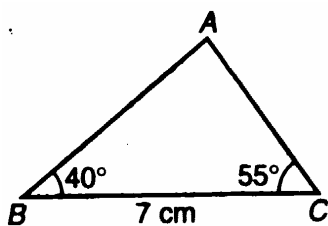


Diagram 10

Calculate the length of AC.

[4.517 cm]

(11) Diagram 11 shows the triangle PQR.

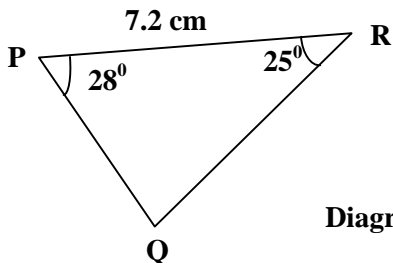


Diagram 11

Calculate the length of PQ.

[3.810 cm]

(12) Diagram 12 shows the triangle DEF.

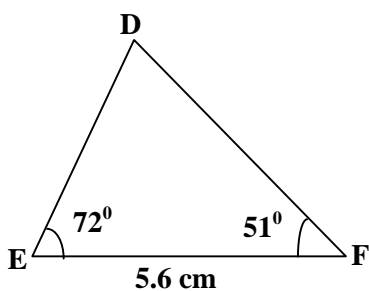


Diagram 12

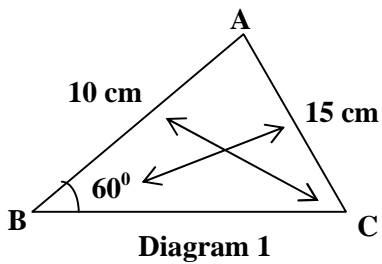
Calculate the length of DE.

[5.189 cm]

Task 3 : Find the unknown angles of a triangle when two sides and a non-included angle are given.

(1) Diagram 1 shows the triangle ABC.

Answer :



Find $\angle ACB$.

$$\frac{\sin C}{10} = \frac{\sin 60^\circ}{15}$$

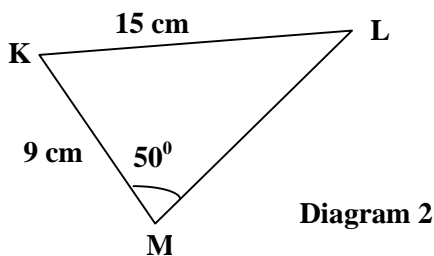
$$\sin C = \frac{10 \sin 60^\circ}{15}$$

$$\sin C = 0.5774$$

$$C = \sin^{-1} 0.5774$$

$$C = 35.27^\circ$$

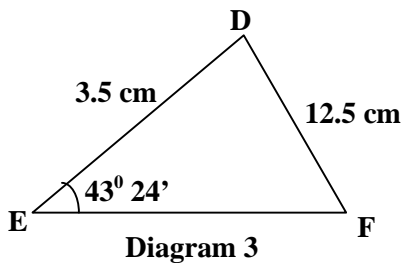
(2) Diagram 2 shows the triangle KLM



Find $\angle KLM$

[27.36°]

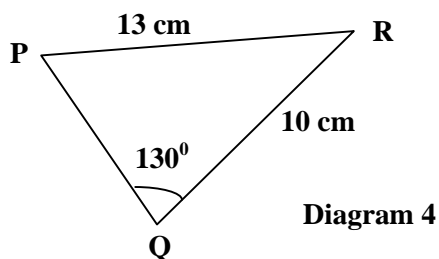
(3) Diagram 3 shows the triangle DEF.



Find $\angle DFE$.

[11.09°]

(4) Diagram 4 shows the triangle PQR.

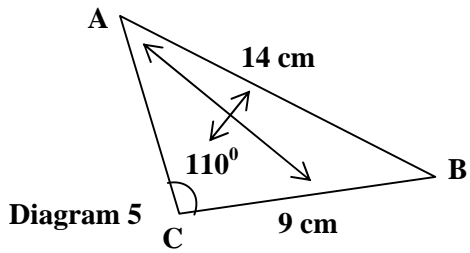


Find $\angle QPR$.

[36.11°]

(5) Diagram 5 shows the triangle ABC.

Answer :



Find $\angle ABC$.

$$\frac{\sin A}{9} = \frac{\sin 110^\circ}{14}$$

$$\sin A = \frac{9 \sin 110^\circ}{14}$$

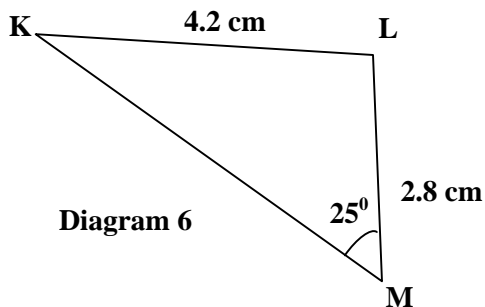
$$\sin A = 0.6041$$

$$A = \sin^{-1} 0.6041$$

$$A = 37.16^\circ$$

$$\begin{aligned} \angle ABC &= 180^\circ - 110^\circ - 37.16^\circ \\ &= 32.84^\circ \end{aligned}$$

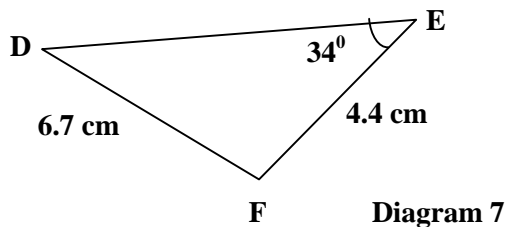
(6) Diagram 6 shows the triangle KLM.



Find $\angle KLM$.

[138.64°]

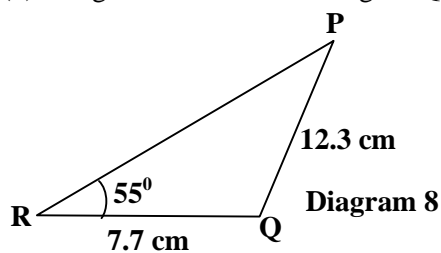
(7) Diagram 7 shows the triangle DEF.



Find $\angle DFE$.

[124.46°]

(8) Diagram 8 shows the triangle PQR.



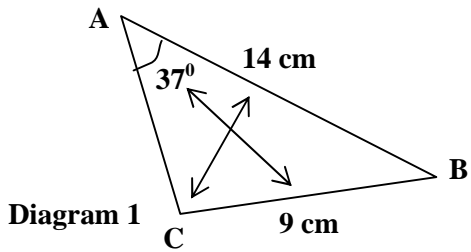
Find $\angle PQR$.

[94.15°]

Task 4 : Find the unknown side of a triangle when two sides and a non-included angle are given.

(1) Diagram 1 shows the triangle ABC.

Answer :



Given that $\angle ACB$ is an obtuse angle, find the length of AC.

$$\frac{\sin C}{14} = \frac{\sin 37^\circ}{9}$$

$$\sin C = \frac{14 \sin 37^\circ}{9}$$

$$\sin C = 0.9362$$

$$C = \sin^{-1} 0.9362$$

$$C = 110.58^\circ$$

$$B = 180^\circ - 110.58^\circ - 37^\circ$$

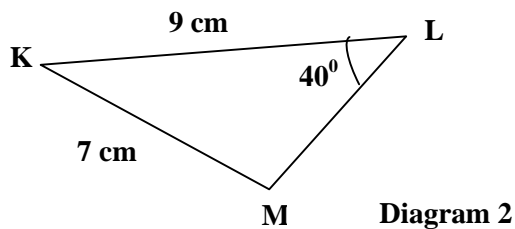
$$= 32.42^\circ$$

$$\frac{AC}{\sin 32.42^\circ} = \frac{9}{\sin 37^\circ}$$

$$AC = \frac{9 \sin 32.42^\circ}{\sin 37^\circ}$$

$$AC = 8.018 \text{ cm}$$

(2) Diagram 2 shows the triangle KLM



Given that $\angle KLM$ is an obtuse angle, find the length of ML.

[2.952 cm]

(3) Diagram 3 shows the triangle DEF.

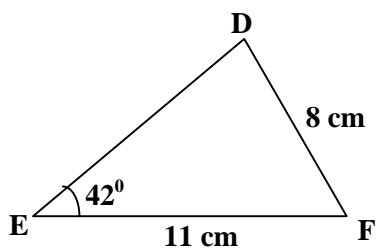


Diagram 3

Given that the value of $\angle EDF$ is greater than 90° , find the length of DE.

[5.040 cm]

(4) Diagram 4 shows the triangle PQR.

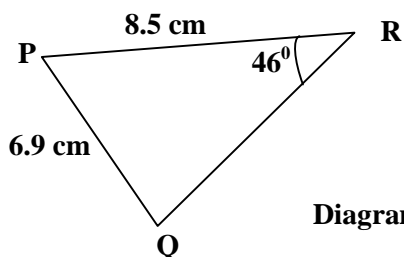


Diagram 4

Given that $\angle PQR$ is an angle in the second quadrant of the cartesian plane, find the length of QR.

[2.707 cm]

(5) Diagram 5 shows the triangle KLM.

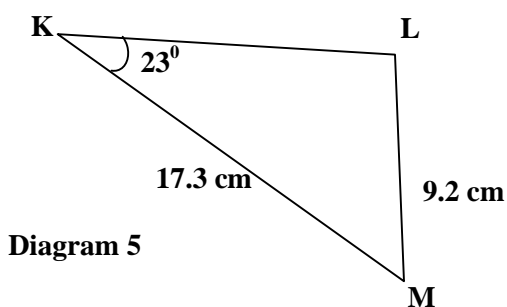


Diagram 5

Given that $\angle KLM$ is an angle in the second quadrant of the cartesian plane, find the length of KL.

[9.686 cm]

SOLUTION OF TRIANGLES

2.2 Use Cosine Rule to find the unknown sides or angles of a triangle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Task 1 : Find the unknown side of a triangle when two sides and an included angle are given.

- (1) Diagram 1 shows the triangle PQR such that PR = 12.3 cm, QR = 16.4 cm and $\angle PRQ = 67^\circ$.

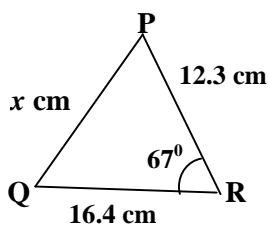


Diagram 1

Find the value of x .

Solution :

$$x^2 = 16.4^2 + 12.3^2 - 2(16.4)(12.3) \cos 67^\circ$$

$$= 262.1$$

$$x = \sqrt{262.61}$$

$$x = 16.21$$

- (2) Diagram 2 shows the triangle PQR such that PQ = 7 cm, QR = 5 cm and $\angle PQR = 75^\circ$.

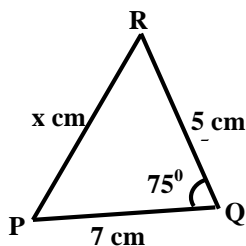


Diagram 2

Find the value of x .

[7.475]

- (3) Diagram 3 shows a triangle with sides 5 cm, 13 cm and an included angle 43° .

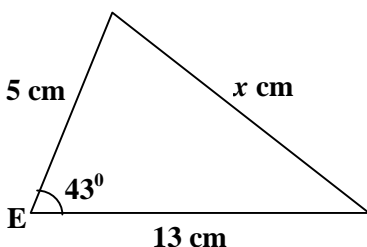


Diagram 3

Find the value of x .

[9.946]

(4) Diagram 4 shows the triangle PQR.

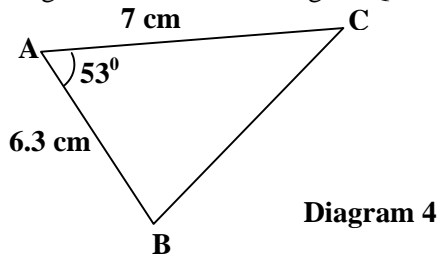


Diagram 4

Find the length of BC.

[5.967 cm]

(5) Diagram 5 shows the triangle KLM.

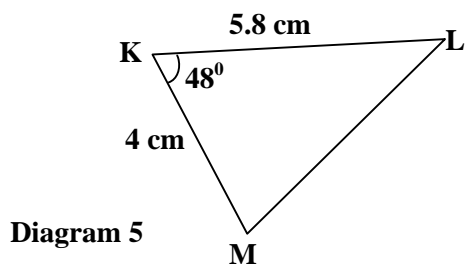


Diagram 5

Find the length of LM.

[4.312 cm]

(6) Diagram 6 shows the triangle PQR.

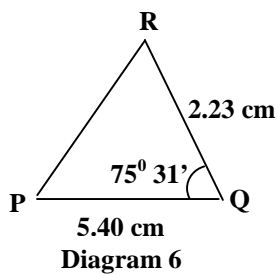


Diagram 6

Find the length of PR.

[5.302 cm]

(7) Diagram 7 shows a triangle with sides 6.21 cm , 10.51 cm and an included angle $36^\circ 39'$.

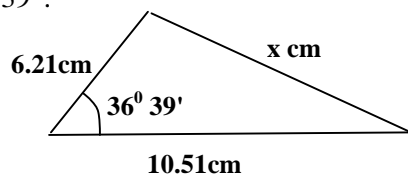


Diagram 7

Find the value of x .

[6.656]

Task 2 : Find the unknown angle of a triangle when three sides are given.

- (1) In Diagram 1, ABC is a triangle where
 AB = 13 cm, AC = 14 cm and BC = 15 cm.

Solution :

$$\cos \angle BAC = \frac{13^2 + 14^2 - 15^2}{2(13)(14)}$$

$$= 0.3846$$

$$\angle BAC = 67.38^\circ$$

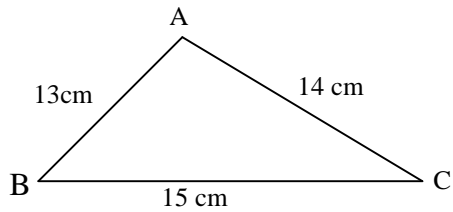


Diagram 1

Find $\angle BAC$.

- (2) Diagram 2 shows a triangle ABC where
 AB = 11 cm, AC = 13 cm and BC = 16 cm.

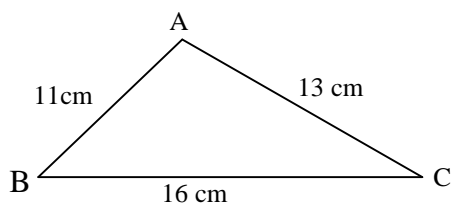


Diagram 2

Find $\angle BAC$.

[83.17°]

- (3) Diagram 3 shows a triangle ABC where
 AB = 13 cm, AC = 16 cm and BC = 17.5 cm.

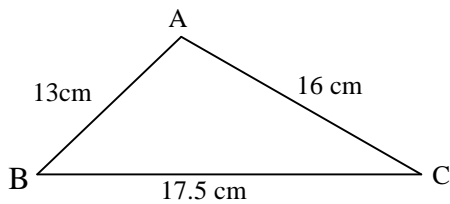


Diagram 3

Calculate $\angle BAC$

[73.41°]

- (4) Diagram 4 shows a triangle ABC where
 AB = 12.67 cm, AC = 16.78 cm and
 BC = 19.97 cm.

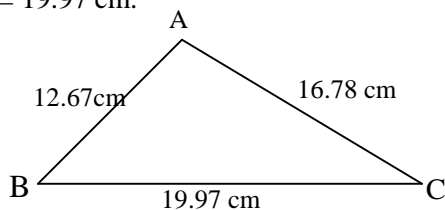


Diagram 4

Calculate $\angle BCA$

[39.17°]

- (5) In Diagram 5, PQR is a triangle such that PR = 6.45 cm, RQ = 2.23 cm and PQ = 5.40 cm.

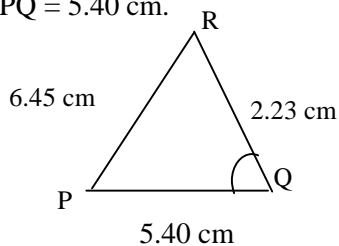


Diagram 5

Find $\angle RQP$.

[108.07°]

- (6) In Diagram 6, PQR is a triangle such that PR = 23.5 cm, RQ = 12.5 cm and PQ = 18.7 cm.

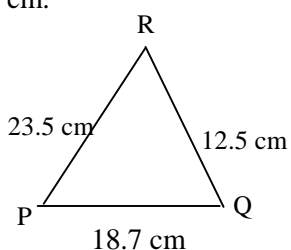


Diagram 6

Calculate the smallest angle in the triangle.

[31.96°]

- (7) For triangle ABC in Diagram 7, AB = 8.56 cm, AC = 11.23 cm and BC = 14.51 cm.

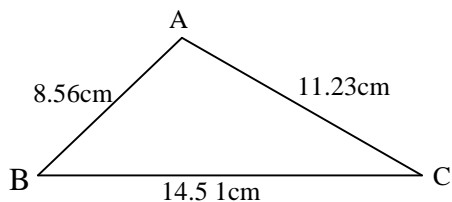


Diagram 7

Calculate the largest angle in the triangle.

[93.33°]

- (8) For triangle ABC in Diagram 8, AB = 13 cm, AC = 16 cm and BC = 17.5 cm.

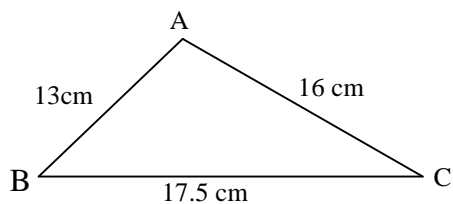


Diagram 8

Calculate the second largest angle in the triangle.

[61.19°]

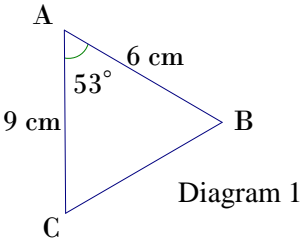
$$\begin{aligned} \text{Area of } \Delta &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} ac \sin B \end{aligned}$$

SOLUTION OF TRIANGLES

3.1 Use the formula $\frac{1}{2} ab \sin C$ or its equivalent to find the area of a triangle.

Task : Find the area of a triangles given in each of the following..

(1) In Diagram 1, ABC is a triangle with $AB = 6 \text{ cm}$, $AC = 9 \text{ cm}$ and $\angle BAC = 53^\circ$. *Solution:*



$$\begin{aligned} \text{Area of } \Delta ABC &= \frac{1}{2} (6)(9) \sin 53^\circ \\ &= 21.56 \text{ cm}^2 \end{aligned}$$

Diagram 1

Find the area of ΔABC

(2) In Diagram 2, ABC is a triangle with $AC = 6 \text{ cm}$, $BC = 5 \text{ cm}$ and $\angle ACB = 78^\circ$.

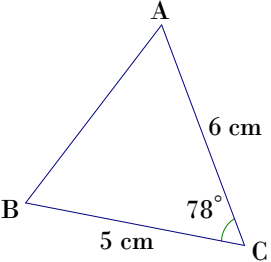


Diagram 2

Find the area of ΔABC . [14.67 cm²]

(3) In Diagram 3, ABC is a triangle with $AC = 6 \text{ cm}$, $BC = 8 \text{ cm}$ and $\angle ACB = 120^\circ$.

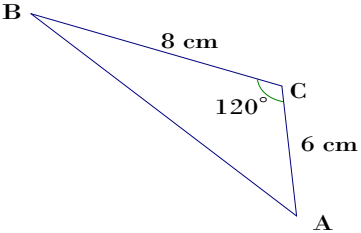


Diagram 3

Find the area of ΔABC . [20.78 cm²]

(4) In Diagram 4, ABC is a triangle with $AC = 6 \text{ cm}$, $BC = 12.5 \text{ cm}$ and the reflex angle $\angle ACB = 250^\circ$.

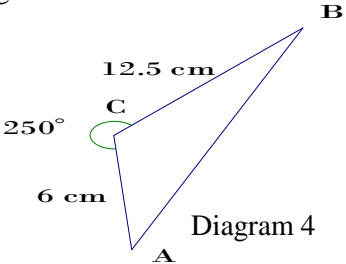


Diagram 4

Find the area of ΔABC . [35.24 cm²]

(5) In Diagram 5, ABC is a triangle such that AB = 12.5 cm, AC = 6 cm and $\angle ACB = 80^\circ$.

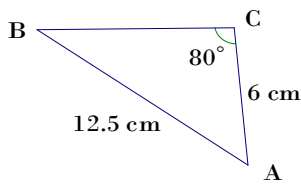


Diagram 5

Find (a) $\angle CBA$,
(b) the area of the triangle.

Solution:

$$(a) \quad \frac{12.5}{\sin 80^\circ} = \frac{6}{\sin \angle CBA}$$

$$\sin \angle CBA = \frac{6 \sin 80^\circ}{12.5} \\ = 0.4727$$

$$\angle CBA = \sin^{-1}(0.4727) \\ = 28.21^\circ$$

$$(b) \quad \angle CAB = 180^\circ - 28.21^\circ - 80^\circ \\ = 71.79^\circ$$

$$\text{Area of } \triangle ABC = \frac{1}{2}(6)(12.5) \sin 71.79^\circ \\ = 35.62 \text{ cm}^2$$

(6) In Diagram 6, ABC is a triangle such that AB = 11 cm, AC = 15 cm and $\angle ACB = 45^\circ 34'$.

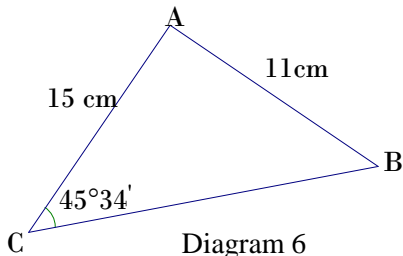


Diagram 6

Find (a) $\angle CBA$,
(b) the area of the triangle.

[(a) 76.83° (b) 69.66 cm^2]

(7) In Diagram 7, ABC is a triangle such that AC = 7 cm, AB = 15 cm and $\angle ACB = 115^\circ 30'$.

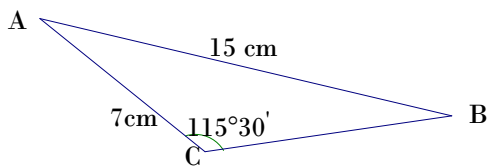


Diagram 7

Find (a) $\angle CBA$,
(b) the area of the triangle

[(a) 24.91° (b) 33.46 cm^2]

(8) In Diagram 8, ABC is a triangle where AB= 15 cm, BC =11 cm and AC=8 cm.

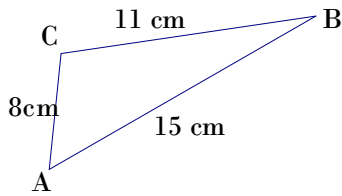


Diagram 8

Find (a) the smallest angle,
(b) the area of ΔABC .

Solution

$$\begin{aligned} \text{(a)} \quad \cos B &= \frac{11^2 + 15^2 - 8^2}{2(11)(15)} \\ &= 0.8545 \\ B &= 31^\circ 30' \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{Area of } \Delta ABC &= \frac{1}{2}(11)(15) \sin 31^\circ 30' \\ &= 42.86 \end{aligned}$$

(9) In Diagram 9, ABC is a triangle where AB= 30 cm, BC =25 cm and AC=20 cm.

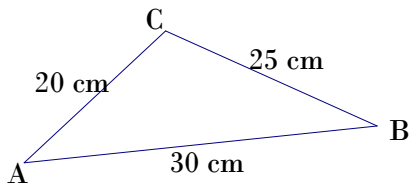


Diagram 9

Find (a) the largest angle,
(b) the area of ΔABC .

[(a) 82.82° (b) 248.04 cm^2]

(10) In Diagram 10, ABC is a triangle where AB = 13 cm, AC = 14 cm and BC= 15 cm.

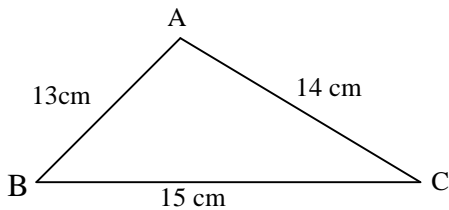


Diagram 10

Find (a) the second largest angle,
(b) the area of ΔABC .

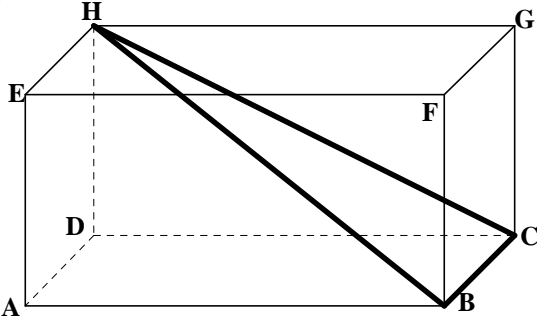
[(a) 59.49° (b) 84.00 cm^2]

SOLUTION OF TRIANGLES

3.2 Solve problems involving three-dimensional objects

Task : Answer all the questions below.

(1)



G Solution:

$$HC = \sqrt{6^2 + 3^2} = 6.708$$

$$BD = \sqrt{6^2 + 4^2} = 7.211$$

$$HB = \sqrt{4.69^2 + 3^2} = 7.810$$

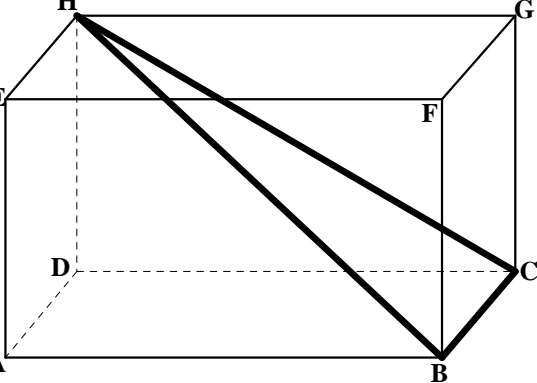
$$\cos \angle BHC = \frac{7.810^2 + 6.708^2 - 4^2}{2(7.810)(6.708)}$$

$$= 0.8589$$

$$\angle BHC = 30.81^\circ$$

The diagram above shows a cuboid with a rectangular base, ABCD. Given that AB = 6cm, BC = 4cm and CG=3 cm. Find $\angle BHC$

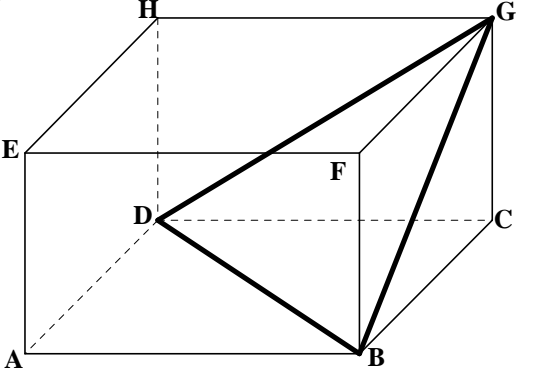
(2)



The diagram above shows a cuboid with a rectangular base ABCD. Given that AB = 16 cm, BC = 4cm and CG=13 cm. Find $\angle BHC$

[10.98°]

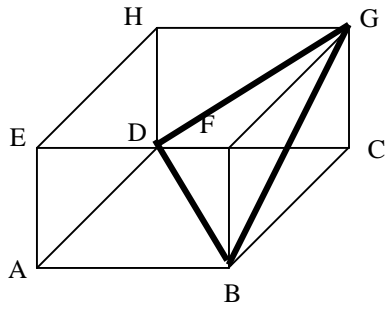
(3)



The diagram above shows a cuboid with a rectangular base ABCD. Given that AB = 6 cm, BC = 4cm and CG = 3 cm. Find $\angle BGD$.

[74.44°]

4.

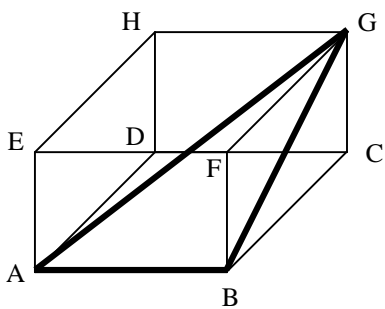


The diagram on the left shows a cuboid with a rectangular base ABCD. Given that $DG=6.1$ cm, $BG=7.2$ cm and $\angle BGD = 41.02^\circ$.

Find the length of BD.

[4.772 cm]

5.



The diagram on the left shows a cuboid with a rectangular base ABCD. Given that $BC = 8.2$ cm, $CG = 6.42$ cm, $AB = 12.03$ cm and $\angle ABG = 110.02^\circ$.

Find the length of AG.

[17.91 cm]

SOLUTION OF TRIANGLES

Further Practice with questions based on SPM format.

Task : Answer all the questions below.

(1) Diagram 1 shows a trapezium $LMNO$.

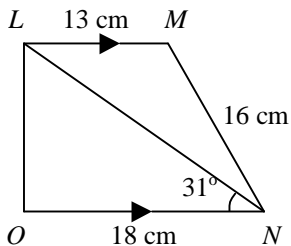


Diagram 1

- Calculate (a) $\angle LNM$,
 (b) the length of LN ,
 (c) the area of $\triangle OLN$.

[(a) 24.74° (b) 25.67 cm (c) 118.99 cm^2]

(2) In Diagram 2, BCD is a straight line.

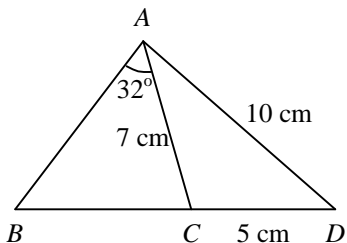


Diagram 2

- Find (a) $\angle ACD$,
 (b) the length of BC ,
 (c) the area of triangle ABD .

[(a) 111.80° (b) 3.769 cm (c) 28.50 cm^2]

(3) In Diagram 3, FGH is a straight line and G is the midpoint of FH .

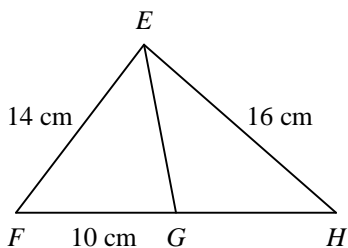


Diagram 3

- Find (a) $\angle EFG$,
 (b) the length of EG ,
 (c) the area of triangle EGH .

[(a) 52.62° (b) 11.23 cm (c) 52.62 cm^2]

(4) Diagram 4 shows a quadrilateral $KLNM$.

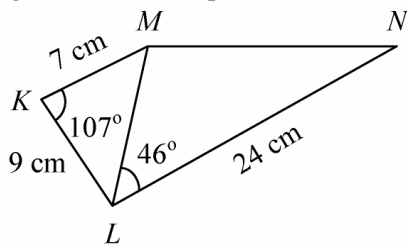


Diagram 4

- Calculate (a) the length of LM ,
 (b) $\angle MNL$,
 (c) the area of quadrilateral $KLNM$.

[(a) 12.92 cm (b) 31.73° (c) 141.65 cm^2]

(5) In Diagram 5, QRS is a straight line.

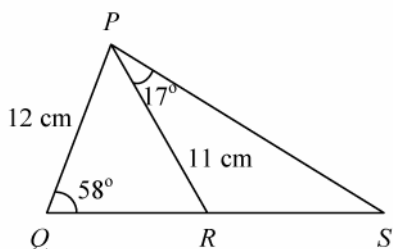


Diagram 5

- Find (a) $\angle QPR$,
 (b) the length of RS ,
 (c) the area of triangle PRS .

[(a) 54.31° (b) 4.157 cm (c) 74.75 cm^2]

(6) In Diagram 6, BCD is a straight line.

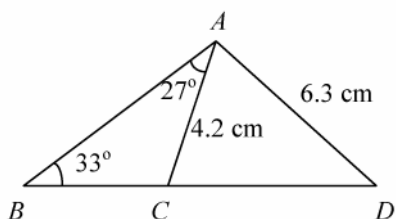


Diagram 6

- Calculate (a) the length of AB ,
 (b) $\angle CAD$,
 (c) the area of triangle ACD .

[(a) 6.678 cm (b) 84.74° (c) 13.17 cm^2]