

MODULE 12
MATEMATIK SPM "ENRICHMENT"
TOPIC : MATRICES
TIME : 2 HOURS

1. (a) The inverse matrix of $\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$ is $m \begin{pmatrix} -4 & n \\ -5 & 3 \end{pmatrix}$

Find the value of m and of n .

- (b) Hence, using matrices, solve the following simultaneous equations :
 $3x - 2y = 8$
 $5x - 4y = 13$

Answer :

(a)

(b)

2. (a) Given that $G = \begin{pmatrix} m & 3 \\ 2 & n \end{pmatrix}$ and the inverse matrix of G is $\frac{1}{14} \begin{pmatrix} 4 & -3 \\ -2 & m \end{pmatrix}$,
find the value of m and of n .

(b) Hence, using matrices, calculate the value of p and of q that satisfies the following equation :

$$G \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \end{pmatrix}$$

Answer :

(a)

(b)

3. (a) Given that $A \begin{pmatrix} -1 & 2 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find matrix A.

(b) Hence, using the matrix method, find the value of r and s which satisfy the simultaneous equations below.

$$\begin{aligned} -r + 2s &= -4 \\ -3r + 5s &= -9 \end{aligned}$$

Answer :

(a)

(b)

4. Given matrix $P = \begin{pmatrix} 4 & 5 \\ 6 & 8 \end{pmatrix}$ and matrix $PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Find the matrix Q.

(b) Hence, calculate by using the matrix method, the values of m and n that satisfy the following simultaneous linear equations :

$$4m + 5n = 7$$

$$6m + 8n = 10$$

Answer :

(a)

(b)

5. Given the matrix P is $\begin{pmatrix} 4 & -3 \\ 8 & -5 \end{pmatrix}$,

(a) Find the matrix Q so that $PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) Hence, calculate the values of h and k , which satisfy the matrix equation:

$$\begin{pmatrix} 4 & -3 \\ 8 & -5 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} -7 \\ -11 \end{pmatrix}$$

Answer :

(a)

(b)

6. (a) Given matrix $M = \begin{pmatrix} k & 6 \\ -4 & 2 \end{pmatrix}$, find the value of k if matrix M has no inverse.

(b) Given the matrix equations

$$\begin{pmatrix} 7 & -6 \\ -5 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{h} \begin{pmatrix} 8 & 6 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

(i) Find the value of h

(ii) Hence, find the value of x and y .

Answer :

(a)

(b)

7. It is given that matrix $P = \begin{pmatrix} 2 & 5 \\ k & -2 \end{pmatrix}$ does not have an inverse matrix.

(a) Find the value of k .

(b) If $k = 1$, find the inverse matrix of P and hence, using matrices, find the values of x and y that satisfy the following simultaneous linear equations.

$$\begin{aligned} 2x + 5y &= 13 \\ x - 2y &= -7 \end{aligned}$$

Answer :

(a)

(b)

8. (a) Find matrix M such that $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} M = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

(b) Using matrices, calculate the values of x and y that satisfy the following matrix equation.

$$\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

Answer :

(a)

(b)

9. (a) Find the inverse of matrix $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$.

(b) Hence, using matrices, calculate the values of d and e that satisfy the following simultaneous equations :

$$\begin{aligned} 2d - e &= 7 \\ 5d - e &= 16 \end{aligned}$$

Answer :

(a)

(b)

10. Given matrix $M = \begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix}$, find

- (a) the inverse matrix of M
- (b) hence, using matrices, the values of u and v that satisfy the following simultaneous equations :

$$\begin{aligned} u - 2v &= 8 \\ 2u + 5v &= 7 \end{aligned}$$

Answer :

(a)

(b)

$$= \frac{1}{2} \begin{pmatrix} 8 & -5 \\ -6 & 4 \end{pmatrix} \quad 1\text{m}$$

$$(b) \quad \begin{pmatrix} 4 & 5 \\ 6 & 8 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \quad 1\text{m}$$

$$\begin{pmatrix} m \\ n \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 8 & -5 \\ -6 & 4 \end{pmatrix} \begin{pmatrix} 7 \\ 10 \end{pmatrix} \quad 1\text{m}$$

$$m = 3 \quad 1\text{m}$$

$$n = -1 \quad 1\text{m}$$

$$5. \quad (a) \quad P = \frac{1}{-20 - (-24)} \begin{pmatrix} -5 & 3 \\ 8 & 4 \end{pmatrix} \quad 1\text{m}$$

$$= \frac{1}{4} \begin{pmatrix} -5 & 3 \\ 8 & 4 \end{pmatrix} \quad 1\text{m}$$

$$(b) \quad \begin{pmatrix} 4 & -3 \\ 8 & -5 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} -7 \\ -11 \end{pmatrix}$$

$$\begin{pmatrix} h \\ k \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -5 & 3 \\ 8 & 4 \end{pmatrix} \begin{pmatrix} -7 \\ -11 \end{pmatrix} \quad 1\text{m}$$

$$= \frac{1}{2} \begin{pmatrix} 2 \\ -100 \end{pmatrix} \quad 1\text{m}$$

$$h = 1 \quad 1\text{m}$$

$$k = -50 \quad 1\text{m}$$

$$6. \quad (a) \quad k = -12 \quad 1\text{m}$$

$$(b) \quad (i) \quad h = 26 \quad 1\text{m}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{26} \begin{pmatrix} 8 & 6 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} -4 \\ 1 \end{pmatrix} \\ = \frac{1}{26} \begin{pmatrix} -26 \\ -13 \end{pmatrix}$$

(ii) 1m

1m

$$x = -1 \quad 1m$$

$$y = -\frac{1}{2} \quad 1m$$

7. (a) $-4 - 5k = 0$ 1m

$$5k = -4$$

$$k = -\frac{4}{5} \quad 1m$$

(b) $\begin{pmatrix} 2 & 5 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -7 \end{pmatrix}$ 1m

$$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{9} \begin{pmatrix} -2 & -5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 13 \\ -7 \end{pmatrix} \quad 1m$$

$$x = -1 \quad 1m$$

$$y = 3 \quad 1m$$

8. (a) $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 2m

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{6-4} \begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ 1m

$$= \frac{1}{2} \begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad 1m$$

$$x = -1 \quad 1m$$

$$y = 2 \quad 1m$$

9. (a) $\frac{1}{-6+5} \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix}$ 1m

$$= \frac{1}{-1} \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix} \quad 1m$$

(b) $\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} 7 \\ 16 \end{pmatrix}$ 1m

$$\begin{pmatrix} d \\ e \end{pmatrix} = \frac{1}{-1} \begin{pmatrix} -3 & 1 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 16 \end{pmatrix} \quad 1m$$

$$= \frac{1}{-1} \begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$d = 5$ 1m

$e = 3$ 1m

10. (a) $\frac{1}{5-(-4)} \begin{pmatrix} 5 & 2 \\ -2 & 1 \end{pmatrix}$ 1m

$$= \frac{1}{9} \begin{pmatrix} 5 & 2 \\ -2 & 1 \end{pmatrix} \quad 1m$$

(b) $\begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$ 1m

$$\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 5 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 7 \end{pmatrix} \quad 1m$$

$$= \frac{1}{9} \begin{pmatrix} 54 \\ -9 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

$u = 6$ 1m

$v = -1$ 1m