5. INDICES AND LOGARITHMS

IMPORTANT NOTES :

UNIT 5.1 Law of Indices

I. $a^m \times a^n = a^{m+n}$

II. $a^m \div a^n = a^{m-n}$

III $(a^m)^n = a^{mn}$

Other Results : $a^0 = 1$, $a^{-m} = \dfrac{1}{a^m}$, $(ab)^m = a^m b^m$

1.	$100^0 =$	$3^{-2} =$	$(3p)^2 =$

5.1.1 "BACK TO BASIC"

BIL	$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$	$(a^m)^n = a^{mn}$
1.	$a^3 \times a^2 = a^{3+2} = a^5$	$a^4 \div a = a^{5-1} = a^4$	$(a^3)^2 = a^{3x2} = a^6$
2.	$2^3 \times 2^4 = 2^{3+4}$ $= 2^{\square}$	$a^3 \div a^5 = a^{3-5} = a^{\square}$ $= \dfrac{1}{a^2}$	$(3^2)^4 = 3^{2x4} = 3^{\square}$
3.	$p^3 \times p^{-4} = p^{3+(-4)}$ $= p^{\square}$ $=$	$p^{-4} \div p^5 = p^{-4-5}$ $= p^{\square}$ $= \dfrac{1}{p^{(\ \ )}}$	$(p^{-5})^2 = p^{-5\,x2} = p^{\square}$ $=$
4.	$2k^3 \times (2k)^3$ $= 2k^3 \times 2^3 \times k^3$ $= (\ \ \ )^{\square}$	$(4a)^2 \div 2a^5 = (4^2a^2) \div (2a^5)$ $= \dfrac{16a^2}{2a^5}$ $=$	$(3x^2)^3 = 3^3 \times x^{2x3}$ $=$

UNIT 5.2 SIMPLE EQUATIONS INVOLVING INDICES

Suggested Steps:

S1 : Use the laws of indices to simplify expression (if necessary)
S2 : Make sure the base is the SAME
S3 : Form a linear equation by equating the indices
S4 : Solve the linear equation

No	Example	Exercise 1	Exercise 2
1.	$3^x = 81$ $3^x = 3^4$ $x = 4$	$2^x = 32$ $x =$	$4^x = 64$ $x =$
2.	$8^x = 16$ $(2^3)^x = 2^4$ $2^{3x} = 2^4$ $3x = 4$ $x = \frac{4}{3}$	$4^x = 32$ $x =$	$27^x = 9$ $x =$
3.	$8^x = 16^{x-3}$ $(2^3)^x = 2^{4(x-3)}$ $2^{3x} = 2^{4x-12}$ $3x = 4x - 12$ $x = 12$	$4^{x+2} = 32^{x-1}$ $x =$	$27^{1-x} = 9^{2x}$ $x =$
4.	$2 \times 8^{2x} = 16^{x+3}$ $2^1 \times (2^3)^x = 2^{4(x+3)}$ $2^{1+3x} = 2^{4x+12}$ $1 + 3x = 4x + 12$ $1 - 12 = 4x - 3x$ $x = -11$	$16 \times 4^{2x-3} = 32^{2-x}$ $x =$	$25^{1-3x} = 5 \times 125^x$ $x =$

UNIT 5.2 LOGARITMS

Do YOU know that ....

If a number N can be expressed in the form N = a^x , then the logarithm of N to the base a is x?

$$N = a^x \Leftrightarrow \log_a N = x$$

$$100 = 10^2 \Leftrightarrow \log_{10} 100 = 2$$

$$64 = 4^3 \Leftrightarrow \log_4 64 = 3$$

$$0.001 = 10^{-3} \Leftrightarrow \log_{10} 0.001 = -3$$

Unit 5.2.1To convert numbers in **index form to *logaritmic form*** and vice-versa.

No.	**Index Form**	**Logaritmic Form**
1.	$10^{2} = 100$	$\log_{10} \mathbf{100} = \mathbf{2}$
2.	$2^3 = 8$	$\log_2 8 = 3$
3.	$p^q = r$	$\log_p r = q$
4.	$10^4 = 10000$	
5.	$a^3 = b$	
6.	$81 = 3^4$	
7.		$\log_p m = k$
8.	$2^x = y$	
9.	$V = 10^x$	
10.		$\log_3 \boldsymbol{x} = \mathbf{y}$
11.		$\log_a y = 2$
12.	$2^5 = 32$	
13.		$\log_3 (xy) = 2$
14.	$10x = y^3$	
15.		$\log_{10} 100y = p$

UNIT 5.2.2 To find the value of a given ***Logaritm***

IMPORTANT : $\log_a a^x = x$

No.	Logaritmic Form	Notes
1.	$\log_{10} \mathbf{1000} = \mathbf{3}$	$10^3 = \mathbf{1000}$ dan $\log_{10} \mathbf{10^3} = \mathbf{3}$
2.	$\log_2 \mathbf{32} = \mathbf{5}$	$2^5 = \mathbf{32}$ dan $\log_2 \mathbf{2^5} = \mathbf{5}$
3.	$\log_{10} \mathbf{0.01} =$	$10^{\square} = \mathbf{0.01}$ dan $\log_{10} \square = \square$
4.	$\log_4 \mathbf{64} =$	$4^{\square} = \mathbf{64}$ dan $\log_4 \square = \square$
5.	$\log_p \sqrt{p} =$	

(REINFORCEMENT)

6.	$\log_p \mathbf{p^8} =$	$\log_a \mathbf{a^2} =$
7.	$\log_m m^{-1} =$	$\log_m \frac{1}{m^2} =$
8.	$\log_a \mathbf{a^{1/3}} =$	$\log_p \mathbf{p^{-5}} =$
9.	$\log_a \frac{1}{a^4} =$	$\log_b \mathbf{b^k} =$
10.	$\log_p \mathbf{(p \times p^2)} =$	$\log_p \mathbf{p}\sqrt{p} =$

UNIT 5.3 Laws of Logaritm

I. $\log_a (xy) = \log_a x + \log_a y$

II. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

III $\log_a x^m = m \log_a x$

Other Results : $\log_a 1 = 0$ (sebab $1 = a^0$)
$\log_a a = 1$ (sebab $a^1 = 1$)

NO.	Examples	Exercises
1.	$\log_a 3pr = \log_a 3 + \log_a p + \log_a r$	(a) $\log_a 2mn$ =
	(b) $\log_a 3aq$ =	(c) $\log_{10} 10yz$ =
	(d) $\log_{10} 1000xy$ =	(e) $\log_2 4mn$ =
2.	$\log_a \frac{p}{q} = \log_a p - \log_a q$	(a) $\log_a \frac{p}{2r} = \log_a p - \log_a 2r$ $= \log_a p - (\log_a 2 + \log_a r)$ =
	(b) $\log_2 \frac{4}{m}$ =	(c) $\log_{10} \left(\frac{10}{kx}\right)$ =
	(d) $\log_{10} \left(\frac{xy}{100}\right)$ =	(e) $\log_a \frac{3a}{m}$ =

UNIT 5.3.2 Aplication of the law : $\log_a x^n = n \log_a x$

3.	Example : $\log_a x^3 = 3 \log_a x$	(a) $\log_a \frac{1}{x^2} = \log_a x^{-2}$ $=$
	(b) $\log_2 (xy^4) = \log_2 x + \log_2 y^4$ $= \log_2 x +$	(c) $\log_2 (4y^4) =$ $=$
	(d) $\log_2 \frac{y^4}{x} =$	(e) $\log_2 \frac{y^4}{8} =$

Reinforcement exercises (Laws of Logaritm)

1.	Example : $\log_{10} 100x^3 = \log_{10} 100 + \log_{10} x^3$ $= \log_{10} 10^2 + 3 \log_{10} x$ $= 2 + 3 \log_{10} x$	(a) $\log_{10} 10000x^5 =$ $=$ $=$
	(b) $\log_2 (\frac{xy^4}{8}) =$ $=$	(c) $\log_p (8p^5) =$ $=$
	(d) $\log_2 \frac{k^2}{4x^3} =$	(e) $\log_4 \frac{y^3}{64} =$

UNIT 5.4 EQUATIONS IN INDICES (Which involves the use of LOGARITM)

I. Equation in the form $a^x = b$

Steps to be followed:

S1 : Take logaritm (to base 10) on both sides.

S2 : Use the law $\log_{10} a^x = x \log_{10} a$.

S3 : Solve the linear equation with the help of a calculator.

No.	Example	Exercise 1	Exercise 2
1.	$3^x = 18$ $\log_{10} 3^x = \log_{10} 18$ $x \log_{10} 3 = \log_{10} 18$ $x = \dfrac{\log_{10} 18}{\log_{10} 3}$ $x =$	$2^x = 9$ $x =$	$7^x = 20$ $x =$
2.	$5^{x+2} = 16$ $\log_{10} 5^{x+2} = \log_{10} 16$ $(x+2) \log_{10} 3 = \log_{10} 18$ $x+2 = \dfrac{\log_{10} 16}{\log_{10} 5}$ $x+2 =$ $x =$	$4^{x+1} = 28$ $x =$	$3^{x-2} = 8$ $x =$
3.	$2^{x+3} = 200$ $\log_{10} 2^{x+3} = \log_{10} 200$ $x+3 =$ $x =$	$7^{1-x} = 2.8$ $x =$	$6^{3x-2} = 66$ $x =$

UNIT 5.5 Change of Base of Logarithms

Formula : $$\log_a x = \frac{\log_b x}{\log_b a}$$

No.	Example	Exercise 1	Exercise 2
1.	$\log_4 8 = \frac{\log_2 8}{\log_2 4}$ $= \frac{3}{2}$	(a) $\log_4 32 = \frac{\log_2 32}{\log_2 4}$ $=$	(b) $\log_{16} 8 =$
	(c) $\log_8 2 =$ $=$	(d) $\log_9 27 =$ $=$	(e) $\log_{81} 9 =$ $=$

(With a calculator) – Change to base 10

1.	$\log_4 9 = \frac{\log_{10} 9}{\log_{10} 4}$ $=$	(a) $\log_5 20 = \frac{\log_{10} 20}{\log_{10} 5}$ $=$	(b) $\log_4 0.8 =$
	(c) $\log_7 2 =$	(d) $\log_9 77 =$	(e) $\log_3 9.6 =$
	(f) $\log_6 2.5 =$	(g) $\log_5 2000 =$	(h) $\log_{12} 6 =$

UNIT 5.6 Aplication of the Laws of Logaritm To Solve Simple Equations involvong logaritms

	EXAMPLE	EVERCISE
C1.	**Solve the equation** $\log_2 (x+1) = 3$. *Answers*: $\log_2 (x+1) = 3$ $x + 1 = 2^3$ $x + 1 = 8$ $x = 7$	**L1. Solve the equation** $\log_2 (x - 3) = 2$. *Jawapan*: **Ans : x = 7**
C2.	**Solve the equation** $\log_{10} (3x - 2) = -1$. *Jawapan*: $3x - 2 = 10^{-1}$ $3x - 2 = 0.1$ $3x = 2.1$ $x = 0.7$	**L2. Solve the equation** $\log_5 (4x - 1) = -1$. **Ans : x = 0.3**
L3.	**Solve the equation** $\log_3 (x - 6) = 2$. **Ans : x = 15**	**L4. Solve the equation** $\log_{10} (1+ 3x) = 2$ **Ans : x = 33**
L5.	**Solve the equation** $\log_3 (2x - 1) + \log_2 4 = 5$. **Ans : x = 14**	**L6. Solve the equation** $\log_4 (x - 2) + 3\log_2 8 = 10$. **Ans : x = 6**
L7.	**Solve the equation** $\log_2 (x + 5) = \log_2 (x - 2) + 3$. **Ans : x = 3**	**L8. Solve the equation** $\log_5 (4x - 7) = \log_5 (x - 2) + 1$. **Ans : x = 3**
L9.	**Solve** $\log_3 3(2x + 3) = 4$ **Ans : x = 12**	**L10 . Solve** $\log_2 8(7 - 3x) = 5$ **Ans : x = 1**

UNIT 5.6.1 To Determine the value of a logarithm without using calculator.

	EXAMPLE	EXERCISE
C1.	**Given $\log_2 3 = 1.585$, $\log_2 5 = 2.322$. Without using a calculator find the value of** (a) $\log_2 15 = \log_2 (3 \times 5)$ $= \log_2 3 + \log_2 5$ $= 1.585 + 2.322$ $=$	**L1. Given $\log_3 5 = 1.465$, $\log_3 7 = 1.771$. Withouf using calculator, evaluate** (a) $\log_3 35 =$ $=$ $=$ $=$
	(b) $\log_2 25 = \log_2 (5 \times 5)$ $=$ $=$ $=$	**(b) $\log_3 49 =$** $=$ $=$ $=$
	(c) $\log_2 0.6 = \log_2 (\frac{3}{5})$ $= \log_2 3 - \log_2 5$ $=$ $=$	**(c) $\log_3 1.4 =$** $=$ $=$ $=$
	(d) $\log_2 10 = \log_2 (2 \times 5)$ $= \log_2 2 + \log_2 5$ $=$ $=$	**(d) $\log_3 21 =$** $=$ $=$ $=$
	(e) $\log_4 5 = \frac{\log_2 5}{\log_2 4}$ $= \frac{2.322}{2}$ $=$	**(e) $\log_9 21 =$** $=$ $=$ $=$
	(f) $\log_5 2 = \frac{\log_2 2}{\log_2 5}$ $= \frac{1}{(\quad\quad)}$ $=$	**(f) $\log_5 3 =$** $= \frac{1}{(\quad\quad)}$ $=$

Enrichment Exercises (SPM Format Questions)

	EXERCISE	EXCERCISE
L1	**Given $\log_3 x$ = m and $\log_2 x$ = n. Find $\log_x 24$ in terms of m and n.** [SPM 2001] [4] **(Ans : 3/n + 1/m)**	**L2. Given $\log_3 x$ = p and $\log_2 x$ = q. Find $\log_x 36$ in terms of m and n.** [4] **(Ans: 2/p + 2/q)**
L3.	**Given $\log_3 x$ = p and $\log_9 y$ = q. Find $\log_3 xy^2$ in terms of p and q.** [SPM 1998] [4] **(Ans: p + 4q)**	**L4. Given $\log_3 x$ = p and $\log_9 y$ = q. Find $\log_3 x^2y^3$ in terms of p and q.** [4] **(Ans: 2p + 6q)**
L5	**Given $\log_5 2$ = m and $\log_5 7$ = p, express $\log_5 4.9$ in terms of m and p. [4]** [SPM 2004] **(Ans: 2p – m – 1)**	**L6. Given $\log_5 2$ = m and $\log_5 7$ = p, express $\log_5 2.8^2$ in terms of m and p. [4]** **(Ans: 2(p + m – 1)**
L7	**Given $\log_2 T - \log_4 V = 3$, express T in terms of V. [4]** [SPM 2003] **(Ans: $T = 8V^{½}$)**	**L8. Given $\log_4 T + \log_2 V = 2$, express T in terms of V. [4]** **(Ans: $16V^{-2}$)**
L9	**Solve** $4^{2x-1} = 7^x$. [4] **(Ans: x = 1.677)**	L10. **Solve** $4^{2x-1} = 9^x$. [4] **(Ans: x = 2.409)**