6. COORDINATE GEOMETRY

Unit 6.1 : To Find the distance between two points ***[BACK TO BASICS]***

A(x_1, y_1) and B(x_2, y_2) : AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Eg. 1 Given two points A(2,3) and B(4,7) Distance of AB = $\sqrt{(4-2)^2 + (7-3)^2}$ $= \sqrt{4 + 16}$ $= \sqrt{20}$ unit.	E1. P(4,5) and Q(3,2) PQ = [$\sqrt{10}$]
E2. R(5,0) and S(5,2) [2]	E3. T(7,1) and U(2,5) [$\sqrt{41}$]
E4. V(10,6) and W(4,2) [$\sqrt{52}$]	E5. X(-4,-1) and Y(-2,1) [$\sqrt{18}$]

More challenging Questions….

E1. The distance between two points A(1, 3) and B(4, k) is 5. Find the possible vales of k. 7, -1	E2. The distance between two points P(-1, 3) and Q(k, 9) is 10. Find the possible values of k. 7, -9
E3. The distance between two points R(-2, 5) and S(1, k) is $\sqrt{10}$. Find the possible vales of k. 6, 4	E4. The distance between two points K(-1, p) and L(p, 9) is $\sqrt{50}$. Find p. p = 0, 6
E5. The distance between two points U(4, -5) and V(2, t) is $\sqrt{20}$. Find the possible vales of t. t =-9, -1	E6. If the distance between O(0, 0) and P(k, 2k) is the same as the distance between the points A(-4, 3) and B(1, -7), find the possible values of k. *k = 5, -5*

Unit 6.2 : Division of a Line Segment

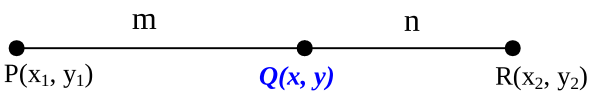
6.2.1 To find the mid-point of Two Given Points.

Formula : Midpoint M $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

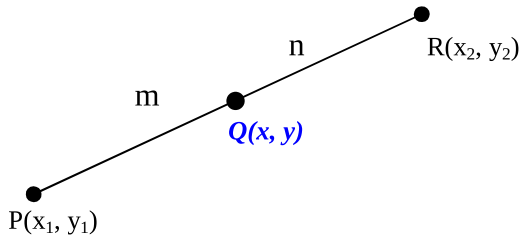
Eg. P(3, 2) and Q(5, 7) Midpoint, M = $\left(\dfrac{3+5}{2}, \dfrac{2+7}{2} \right)$ = $(4, \dfrac{9}{2})$	E1 P(-4, 6) and Q(8, 0) (2, 3)
E2 P(6, 3) and Q(2, -1) (4, 1)	E3 P(0,-1), and Q(-1, -5) (- ½ , -3)

6.2.2 Division of a Line Segment

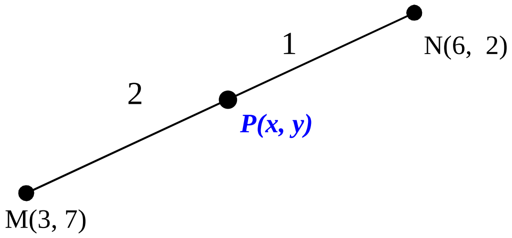
Q divides the line segment PR in the ratio *PQ : QR* = m : n. P(*x*, *y*), R(*x*, *y*)



Q (*x*,*y*) = $\left(\dfrac{nx_1 + mx_2}{m+n}, \dfrac{ny_1 + my_2}{m+n} \right)$



(NOTE : Students are strongly advised to sketch a line segment before applying the formula)

Eg1. The point P internally divides the line segment joining the point M(3,7) and N(6,2) in the ratio 2 : 1. Find the coordinates of point P.  P = $\left(\dfrac{1(3)+2(6)}{2+1}, \dfrac{1(7)+2(2)}{2+1} \right)$ = $\left(\dfrac{15}{3}, \dfrac{11}{3} \right)$ = $\left(5, \dfrac{11}{3} \right)$	E1. The point P internally divides the line segment joining the point M (4,5) and N(-8,-5) in the ratio 1 : 3. Find the coordinates of point P. $\left(1, \dfrac{5}{2} \right)$

More Exercise : The Ratio Theorem

(NOTE : Students are strongly advised to sketch a line segment before applying the formula)

E1. R divides PQ in the ratio 2 : 1. Find the coordinates of R if (a) P(1, 2) and Q(-5, 11) (b) P(-4, 7) and Q(8, -5) (a) (-3, 8) (b) (4 , -1)	E2. P divides AB in the ratio 3 : 2. Find the coordinates of P if (c) A(2, -3) and B(-8, 7) (d) A(-7, 5) and B(8, -5) (a) (-4, -3) (b) (2 , -1)
E3. M is a point that lies on the straight line RS such that 3RM = MS. If the coordinates of the points R and S are (4,5) and (-8,-5) respectively, find the coordinates of point M. 3RM = MS $\frac{RM}{MS} = \frac{1}{3}$, RM : MS = 1 : 3 Ans : $\left(1, \frac{5}{2}\right)$	E4. P is a point that lies on the straight line TU such that 3TP = 2PU. If the coordinates of the points T and U are (-9,7) and (1,-3) respectively, find the coordinates of point P. (-5, 3)
E5. The points P(3, p), B(-1, 2) and C(9,7) lie on a straight line. If P divides BC internally in the ratio m : n , find (a) m : n , (b) the value of p. (a) 2 : 3 (b) p = 4	E6. R(*x*, *y*) , divides the points P(2k, – k) and Q(2*x*, 4*y*) in the ratio 3 : 5. Express *x* in terms of *y*. (Ans : x = 4y)

Unit 6.3 To Find Areas of Polygons

Area of a polygon = $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & \ldots & x_1 \\ y_1 & y_2 & y_3 & \ldots & y_1 \end{vmatrix}$

Note : The area found will be positive if the coordinates of the points are written in the anti-clockwise order, and negative if they are written in the clock-wise order.

Example 1 : Calculate the area of a triangle given :

E1. P(0, 1), Q(1, 3) and R(2,5) Area of Δ PQR = $\frac{1}{2}\begin{vmatrix} 0 & 1 & 2 & 0 \\ 1 & 3 & 5 & 1 \end{vmatrix}$ = = 12 unit ^{2}	1. P(2,3), Q(5,6) and R(-4,4) Area of Δ PQR = $\frac{17}{2}$ unit ^{2}
2. The coordinates of the triangle ABC are (5, 10), (2,1) and (8, k) respectively. Find the possible values of k, given that the area of triangle ABC is 24 units ^{2}. k = 3 , 35	3. The coordinates of the triangle RST are (4, 3), (-1, 1) and (t, -3) respectively. Find the possible values of t , given that the area of triangle RST is 11 units ^{2}. t = 0 , -22

ii) Area of a quadrilateral = $\frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$

1. P(1,5), Q(4,7), R(6,6) and S(3,1). Area of PQRS = = 8 unit ^{2}	2. P(2, -1), Q(3,3), R(-1, 5) and S(-4, -1). [27]

Note : If the area is zero, then the points are collinear.

1. Given that the points P(5, 7), Q(4, 3) and R(-5, k) are collinear, find the value of k. k= 33	2. Show that the points K(4, 8), L(2, 2) and M(1, -1) are collinear.

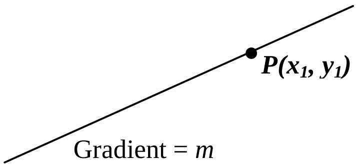
Unit 6.4 : Equations of Straight Lines

The Equation of a Straight line may be expressed in the following forms:

i) The general form : $ax + by + c = 0$

ii) The gradient form : $y = mx + c$; $m =$ *gradient* , $c =$ *y-intercept*

iii) The intercept form : $\frac{x}{a} + \frac{y}{b} = 1$, $a =$ *x-intercept* , $b =$ *y-intercept*

a) If given the gradient and one point: $y - y_1 = m(x - x_1)$  	Eg. Find the equation of a straight line that passes through the point (2,-3) and has a gradient of $\frac{1}{4}$. $y - y_1 = m(x - x_1)$ $y - (-3) = \frac{1}{4}(x - 2)$ $4y = x - 14$
E1. Find the equation of a straight line that passes through the point (5,2) and has a gradient of -2. y = -2x + 12	E2. Find the equation of a straight line that passes through the point (-8,3) and has a gradient of $\frac{3}{4}$. 4y = 3x + 36
b) If two points are given : Note : You may find the gradient first, then use either (a) $y = mx + c$ Or (b) $y - y_1 = m(x - x_1)$ Or (c) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	Eg. Find the equation of a straight line that passes through the points (-3, -4) and (-5,6) $\frac{y - (-4)}{x - (-3)} = \frac{6 - (-4)}{-5 - (-3)}$ $y = -5x - 19$
E1. Find the equation of a straight line that passes through the points (2, -1) and (3,0) y = x - 3	E2. Find the equation of a straight line that passes through the points (-4,3) and (2,-5) 4x + 3y +7 = 0

c) *The x-intercept and the y-intercept are given:*

$m = -\left(\dfrac{y - \text{int}\,ercept}{x - \text{int}\,errcept}\right)$

Equation of Straight Line is :

$$\frac{x}{a} + \frac{y}{b} = 1$$

Note : Sketch a diagram to help you !

At the x-axis, y = 0
At the y-axis, x = 0

E1. The x-intercept and the y-intercept of the straight line PQ are 4 and -8 respectively. Find the gradient and the equation of PQ.

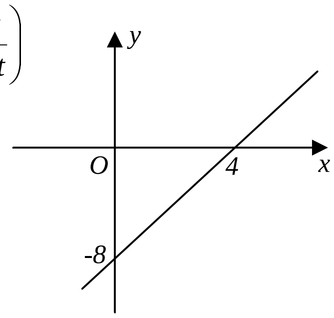
$m_{PQ} = -\left(\dfrac{y - \text{int}\,ercept}{x - \text{int}\,errcept}\right)$

$= -\left(\dfrac{-8}{4}\right)$

$= 2$

Equation : $\dfrac{x}{4} + \dfrac{y}{-8} = 1$

$y = 2x - 8$



E2. The x-intercept and the y-intercept of the straight line PQ are -6 and 3 respectively. Find the gradient and the equation of PQ.

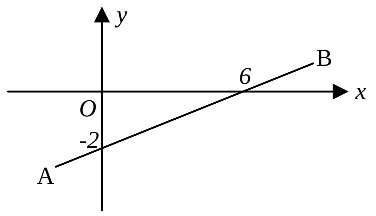
2y = x+6

E3. The x-intercept of a straight line AB is -5 and its gradient is -3. Find the y-intercept of the straight line AB and the equation of AB.

3x + 5y +15 = 0

Extra Vitamins for U……

1. Find the gradient and the equation of AB.

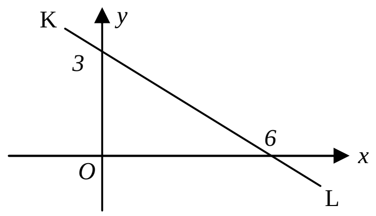


x – 3y = 6

2. The x-intercept of a straight line RS is -2 and its gradient is 3. Find the y-intercept of the straight line RS and the equation of RS.

y = 3x + 6

3. Find the equation of KL in the intercept form.



$\frac{x}{6} + \frac{y}{3} = 1$

4. Find the equation of the line which connects the origin and the point S (-2, 6).

y = – 3x

5. For Q3 above, write down the equation of KL in the general form.

x + 2y – 6 = 0

6. Write down the equation of the straight line which passes through the points P(3, 2) and Q (3, 8).

[x = 3]

Unit 6.5 Parallel Lines and Perpendicular lines

6.5.1	**Parallel lines,** $m_1 = m_2$
6.5.2	**Perpendicular lines,** $m_1 m_2 = -1$

Unit 6.5.1 Determine whether each of the following pairs of lines are parallel.

Eg. y = 3x – 2 and 3x – y = 4 y = 3 x – 2 , m_1 = 3 3x – y = 4 y = 3x – 4 , m_2 = 3 Since $m_1 = m_2$, ∴ the two line are parallel .	1. y = 2x +5 and 4x + 2y = 5 N
2. 3x – 3y = 7 and 6x + 6y = – 5 N	3. 2x – 3y = 5 and 6y = 4x + 9 Y
4. x – 3y = 12 and 6y = 3 + 2x Y	5. $\frac{x}{3} - \frac{y}{2} = 4$ and 8y = 6x - 3 N

Unit 6.5.2 Determine whether each of the following pairs of lines are perpendicular.

Eg. y = 3x – 2 and x + 3y = 4 y = 3 x – 2 , m_1 = 3 x + 3y = 4 3y = – x + 4 $y = -\frac{1}{3}x + \frac{4}{3}$, $m_2 = -\frac{1}{3}$ Since $m_1 . m_2 = 3 \times -\frac{1}{3} = -1$, ∴ The two given lines are perpendicular .	1. y = 2x +5 and 4x + 2y = 9 N
2. 3y = 2x – 2 and 2x + 3y = 1 N	3. x – 3y = 2 and 6x + 2y = 5 Y
4. 6y = 2 - 3x and $\frac{x}{3} - \frac{y}{6} = 4$ Y	5. $\frac{x}{3} - \frac{y}{4} = 1$ and 8y + 6x – 3 = 0 Y

6.5.2 Applications ($m_1.m_2 = -1$)

Ex.1 (SPM 2004). Diagram 1 shows a straight line PQ with the equation $\frac{x}{2} + \frac{y}{4} = 1$. Find the equation of the straight line perpendicular to PQ and passing through the point Q.

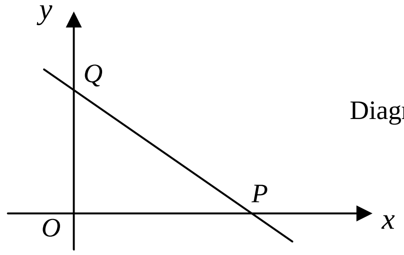


Diagram 1

Answer:

y = ½ x + 4

Ex.2. Diagram 2 shows a straight line PQ with the equation $\frac{x}{6} + \frac{y}{2} = 1$. Find the equation of the straight line perpendicular to PQ and passing through the point P.

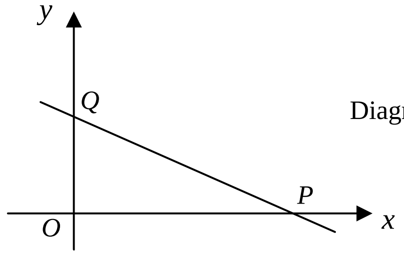


Diagram 2

Answer:

y = 3x – 18

Ex.3 Diagram 3 shows a straight line RS with the equation x + 2y = 6. Find the equation of the straight line perpendicular to RS and passing through the point S.

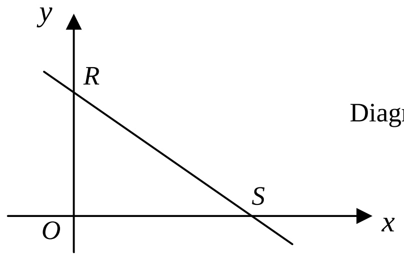


Diagram 3

Answer:

y = 2x – 12

Ex.4. Diagram 4 shows a straight line AB with the equation 2x – 3y = 6. Find the equation of the straight line perpendicular to AB and passing through the point B.

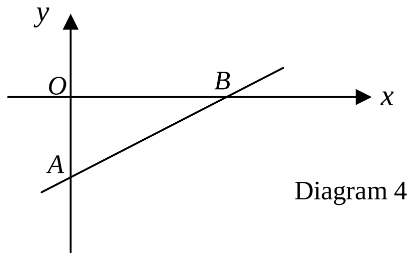


Diagram 4

Answer:

2y = 3x – 9

6.5.2 Applications ($m_1.m_2 = -1$) – more exercises

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Ex.5 Diagram 5 shows a straight line PQ with the equation 4x + 3y = 12. Find the equation of the straight line perpendicular to RS and passing through the midpoint of RS.

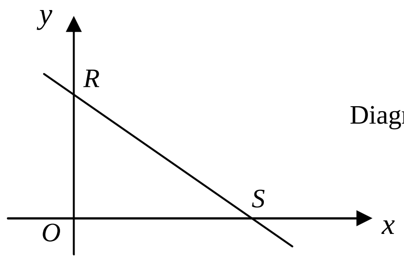

Diagram 5

Answer:

4x+3y = 8

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Ex.6. Diagram 6 shows a straight line AB with the equation $\frac{x}{4} - \frac{y}{6} = 1$. Find the equation of the perpendicular bisector of the line AB.

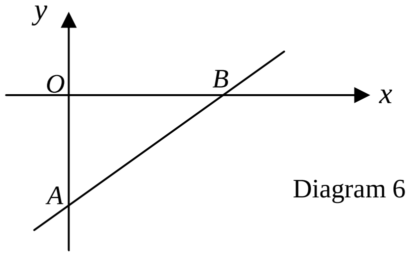

Diagram 6

Answer:

2x + 3y = 6

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Ex.7. Find the equation of the straight line that passes through the point (1, 2) and is perpendicular to the straight line x + 3y +6 = 0.

y = 3x – 1

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Ex.8 Find the equation of the straight line that passes through the point (3, 0) and is perpendicular to the straight line 3x – 2y = 12.

2x+3y = 6

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Ex.9 Find the equation of the straight line that passes through the origin O and is perpendicular to the straight line that passes through the points P(1, – 1) and Q(-3,7).

y = ½ x

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Ex. 10 Find the equation of the straight line that passes through the point (-2,4) and is perpendicular to the straight line which passes through the origin O and the point (6, 2).

y = -3x

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Unit 6.6 Equation of a Locus

Note : Students MUST be able to find distance between two points [using Pythagoras Theorem]

TASK : To Find the equation of the locus of the moving point P such that its distances of P from the points Q and R are equal.

Eg 1. Q(6, -5) and R(1,9)

Let P = (x,y), then PQ = PR

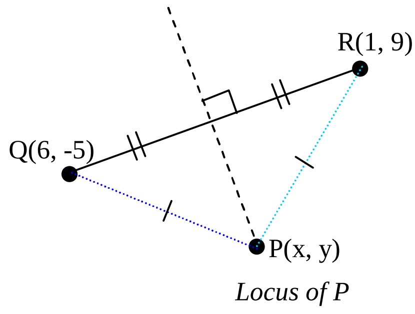
$$\sqrt{(x-6)^2+(y-(-5))^2}=\sqrt{(x-1)^2+(y-9)^2}$$

Square both sides to eliminate the square roots.

$$\left(x-6\right)^2+(y+5)^2=\left(x-1\right)^2+(y-9)^2$$

$$x^2-12x+36+y^2+10y+25=x^2-2x+1+y^2-18y+81$$

$$10x-28y+21=0$$



E1. Q(2,5) and R(4,2) *4x – 6y+9 =0*	E2. Q(-3, 0) and R(6, 4) *18x + 8y = 43*
E3. Q(2, -3) and R(-4, 5) *3x – 4y + 3 = 0*	E4. Q(6, -2) and R(0, 2) *3x – 2y – 9 = 0*

More challenges…….

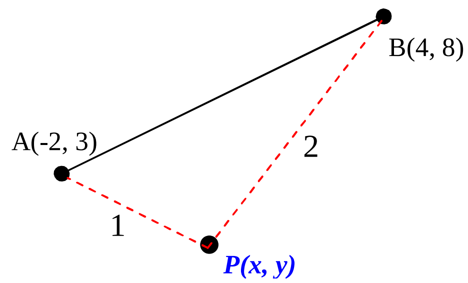
E5. Given two points A(3, 2) and B(7, -4). Find the equation of the perpendicular bisector of AB. *3y =2x - 13*	E6. Given two points P(4, 10) and QB(-6, 0). Find the equation of the the perpendicular bisector of PQ. *x + y = 4*

TASK : To find the equation of the locus of the moving point P such that its distances from the points A and B are in the ratio m : n

(Note : Sketch a diagram to help you using the distance formula correctly)

Eg 1. A(-2,3), B(4,8) and m : n = 1: 2

Let P = (x, y)



$$\frac{LP}{PM} = \frac{1}{2}$$

$$2LK = KM$$

$$2\sqrt{(x-(-2))^2 + (y-3)^2} = \sqrt{(x-4)^2 + (y-8)^2}$$

$$(2)^2\left(\sqrt{(x+2)^2 + (y-3)^2}\right)^2 = \left(x-4)^2 + (y-8\right)^2$$

$$4(\left(x+2)^2 + (y-3\right)^2) = \left(x-4)^2 + (y-8)^2\right.$$

$$4x^2 + 16x + 16 + 4y^2 - 24y + 36 = x^2 - 8x + 16 + y^2 - 16y + 64$$

$3x^2 + 3y^2 + 24x - 8y - 28 = 0$ is the equation of locus of P.

E1. A(1, 5), B(4, 2) and m : n = 2 : 1 *$x^2+y^2 - 10x - 2y + 19 = 0$*	E2. A(-3, 2), B(3, 2) and m : n = 2 : 1 *$x^2+y^2 - 10x - 6y + 13 = 0$*
E3. A(1, 3), B(-2, 6) and m : n = 1 : 2 *$x^2+ y^2 - 3x - 3y = 0$*	E4. A(5, -2), B(-4, 1) and m : n = 1 : 2 *$x^2+ y^2 - 16x + 6y + 33 = 0$*
E5. P(-1, 3), Q(4, -2) and m : n = 2 : 3 *$x^2+y^2+ 10x - 14y + 2 = 0$*	E6. A(1, 5), B(-4, -5) and m : n = 3 : 2 *$x^2+y^2 + 16x +26y + 53 = 0$*

SPM FORMAT QUESTIONS

1. (2003) The equations of two straight lines are $\frac{y}{5} + \frac{x}{3} = 1$ and 5y = 3x + 24. Determine whether the lines are perpendicular to each other.

[Y]

2. The equations of two straight lines are $\frac{x}{3} - \frac{y}{2} = 4$ and 3y = 2x + 6. Determine whether the lines are perpendicular to each other.

[N]

3.(2004) Diagram 4 shows a straight line PQ with the equation $\frac{x}{2} + \frac{y}{3} = 1$. Find the equation of the straight line perpendicular to PQ and passing through the point Q.

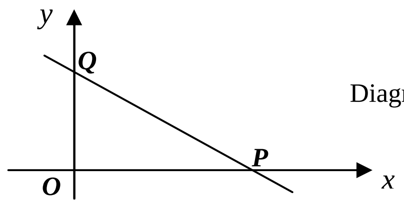


Diagram 4

[$y = \frac{2}{3}x + 3$]

4. Diagram 5 shows a straight line RS with the equation $\frac{x}{6} + \frac{y}{4} = 1$. Find the equation of the straight line perpendicular to RS and passing through the point S.

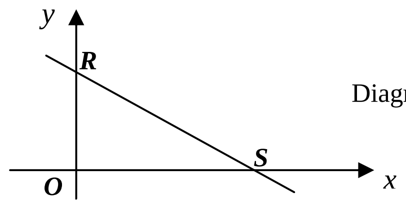


Diagram 5

[2y = 3x - 18]

5. (2005) The following information refers to the equations of two straight lines, JK and RT, which are perpendicular to each other.

> *JK : y = px + k*
> *RT : y = (k – 2)x + p*
> *where p and k are constants.*

Express p in terms of k.

$p = \frac{1}{2-k}$

6. The following information refers to the equations of two straight lines, PQ and RS, which are perpendicular to each other.

> *PQ : px + y = k*
> *RS : y = (2k –1)x + p*
> *where p and k are constants.*

Express p in terms of k.

$p = \frac{1}{2k-1}$

7. (2006) Diagram 5 shows the straight line AB which is perpendicular to the straight line CB at the point B.

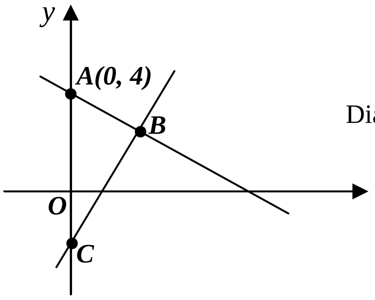


Diagram 5

The equation of CB is $y = 2x - 1$.
Find the coordinates of B.

(2, 3)

8. Diagram 6 shows the straight line PQ which is perpendicular to the straight line RQ at the point Q.

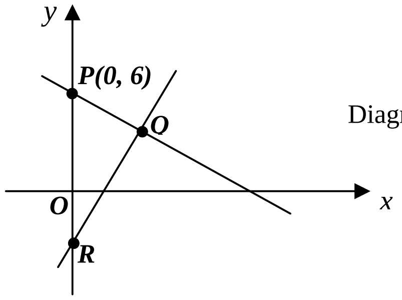


Diagram 6

The equation of QR is $x - y = 4$.
Find the coordinates of Q

Q(5, 1)

9. (2004) The point A is (-1, 2) and B is (4, 6). The point P moves such that PA : PB = 2 : 3. Find the equation of locus of P. [3 marks]

$[5x^2+5y^2+50x+12y+163=0]$

10. The point R is (3, -5) and S is (0, 1). The point P moves such that PR : PS = 2 : 1. Find the equation of locus of P. [3 marks]

$[x^2+y^2+2x-6y-10=0]$

11. The point A is (8, -2) and B is (4, 6). Find the equation of the perpendicular bisector of AB. [3 marks]

$2y = x - 2$

12. The point R is (2, -3) and S is (4, 5). The point P moves such that it is always the same distance from R and from S. Find the equation of locus of P. [3 marks]

$x+4y = 7$